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RESEARCH ARTICLE

How nonlinear benchmark in delegation contract can affect asset price and price informativeness

Jiliang Sheng[1] · Yanyan Yang[1] · Xiaoting Wang[2] · Jun Yang[3]

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Abstract
Delegation contracts with conventional linear benchmarking cannot motivate institutions to acquire information, which deteriorates price informativeness and increases return volatility. This study investigates performance-based contracts in which the benchmark is a nonlinear (quadratic) function of the benchmark portfolio return. In a unified model incorporating both information acquisition and investment decisions, we show that delegation contracts with the nonlinear benchmark can overcome the weakness of conventional benchmarked contracts. Specifically, they can incentivize information acquisition, enhance price informativeness, lower return volatility, and, when penalty intensity is relatively low, increase institutions' expected utility and reduce fixed delegation costs. The impact of the contract's incentive component on the equilibrium price and price informativeness depends on the average incentive slope. Further analysis finds that delegated investment by informed institutional investors can improve price informativeness. This effect is more pronounced under nonlinear benchmarked contracts than under non-benchmarked or linear benchmarked contracts.

Keywords Performance-based contract · Nonlinear benchmark · Information acquisition incentive · Price informativeness

JEL Classification D82 · D86 · G11 · G23 · M52

1 Introduction

Institutional investors prevail in asset management markets due to their professional expertise and technological advantages. Berk and van Binsbergen (2015) report that

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✉ Jun Yang
jun.yang@acadiau.ca

[1] School of Statistics, Jiangxi University of Finance and Economics, Nanchang, China

[2] Department of Economics, Acadia University, Wolfville, Canada

[3] F.C. Manning School of Business Administration, Acadia University, Wolfville, NS B4P 2R6, Canada

fund managers can create an average incremental value of $3.2 million per year by leveraging their professional know-how. Given the difference in preference and risk appetites between institutional investors and their clients, a research hotspot is how to incentivize institutions while keeping moral hazard in check (Brown and Davies 2017). However, information acquisition is expensive for institutional investors (Duraj and Lin 2022). The key issue is to balance the interests of investors and institutions while maintaining or even improving price informativeness. Higher price informativeness indicates better information transparency, which helps reduce information acquisition costs and improve market informational efficiency. Besides regulatory policies,[1] asset management contracts are a primary mechanism for motivating institutions to pursue information and thus improve price informativeness. Studying the effects of different forms of delegation contracts in asset management has important theoretical and practical significance for reducing financial risks and improving market efficiency.

Benchmarked contracts, also known as performance-based contracts, explicitly evaluate the investment performance of an institution against a benchmark portfolio and issue reward or penalty accordingly. When examining compensation contracts of portfolio manager in U.S. mutual fund industry, Ma et al. (2019) find that the bonus component of managers' compensation is explicitly tied to the fund's investment performance for 79.0% of the sample funds. Performance-based contracts integrate the carrot-and-stick incentivizing mechanisms. Sotes-Paladino and Zapatero (2022) argue that compensation contracts with linear benchmarking are suboptimal in incentivizing information acquisition and improving price informativeness, but the penalty component can effectively balance the difference in risk appetite between investors and institutions. Investors can benefit from institutions' information acquisition without increasing costs.

The vast majority of the literature on the subject is on linear benchmarking due to its intuitiveness. Breugem and Buss (2019) find that linear benchmarking has no incentive effect on institutions' information acquisition. Compared to the case without performance-based evaluation, linear benchmarking actually reduces institutional investors' effort in information acquisition and lowers price informativeness, thus causing a decline in market informational efficiency. Basak and Pavlova (2013) and Buffa et al. (2014) find that linear benchmarking in compensation contracts makes institutions to peg their investment to the benchmark portfolio, resulting in an unfounded increase in risky assets' prices. The root reason for such price distortion is that, as the benchmarking is linear, institutions can avoid potential penalty by holding mostly the benchmark portfolio.

Besides leading to insufficient effort in information acquisition and inferior price informativeness, linear benchmarking has another major drawback. Under conventional performance-based contracts, as long as an institution's investment performance surpasses the benchmark, it will be rewarded even if its investment return is negative. However, this is often deemed unreasonable or even unacceptable to highly risk averse investors. In addition, excessive risk taking or high-leverage investment strategies by

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[1] Policy-related issues that have been examined in literature include information diversity and complementarities (Goldstein and Yang 2015), types of disclosures (Goldstein and Yang 2019; Lemus and Temnyalov 2023), prevention of the crowding out of private information by public information (Han et al. 2016), and short selling restrictions (Nezafat et al. 2017).

institutions might in extreme scenarios make the value of wealth under management negative. Conventional performance-based contracts do not impose constraints and penalties on negative returns or wealth values. Cuoco and Kaniel (2011), Lioui and Poncet (2013), and Escobar-Anel et al. (2020) argue that when the wealth value managed by institutions is negative, the utility to investors is negative infinity. Therefore, loss loathing investors have a strong desire to modify the conventionally benchmarked contracts, requiring institutions to exploit their information advantage and expertise to deliver a positive investment return. Institutions should be punished in case of negative returns.

To address the challenge of insufficient incentive for information acquisition and reduced price informativeness caused by linear benchmarking, as well as to incentivize institutional investors to pursue positive returns, this study turns to nonlinear benchmarking and proposes a novel contract whose benchmark is a quadratic function of the benchmark portfolio return. It effects are investigated and compared to those of the linear benchmarked contract in terms of incentive for information acquisition, price informativeness, and equilibrium asset price.

Among the few studies involving nonlinear benchmarking or nonlinear contracts, Bhattacharaya and Pfleiderer (1985) set the penalty term as the square of the difference between the predicted and actual payoff of the risky asset in order to motivate institutions to provide accurate information. The higher the accuracy of the information acquired by the institution, the smaller the penalty. Stoughton (1993) finds that the contracts with nonlinear penalty component proposed by Bhattacharaya and Pfleiderer (1985) can better incentivize information acquisition than linear benchmarked contracts. Brennan and Cao (1996) argue that the Pareto optimal allocation should include a quadratic option. Cao (1999) believes that allocation schemes involving quadratic options can stimulate information acquisition and reduce price volatility. Cuoco and Kaniel (2011) reveal that nonlinear penalty terms can mitigate price distortion caused by relative performance contracts. Cvitanić and Xing (2018) demonstrate that the optimal contract should encourage agents to take specific risk of individual assets by rewarding the manager for return in excess of a benchmark portfolio value, and for quadratic deviation thereof.

In contrast to the literature, this paper studies from the perspective of nonlinear benchmarks. In order to motivate institutions to obtain a positive investment return and solve the sub-optimal problem of linear benchmarking in institutional information acquisition, the nonlinear benchmark proposed in this paper is a quadratic function of the benchmark portfolio return. As the party with informational advantage, the institution bears the tail risk brought by negative investment returns. The non-linear benchmark makes it impossible for institutions to adjust their investment strategies to directly hedge against the performance-induced penalty. In order to avoid or reduce the penalty brought by sub-par performance, institutions have to strive to obtain more accurate information regarding price movements and execute proper investment strategies. When the proportion of institutions under the quadratic benchmark increases, price informativeness improves and return fluctuation falls. This paper verifies the above intuition with a theoretical model.

Besides exploring the effects of non-linear benchmarking, this paper also studies the effects of the incentive slope from the perspective of institutional investors both individually and collectively.[2] Basak and Pavlova (2013) believe that institutionalization causes excessive holding of the benchmark portfolio by institutions and thus leads to higher prices of the assets in the benchmark. Huang et al. (2020) argue that institutionalization introduces more investors with informational advantages to the market, thereby enhancing price informativeness. Is improved price informativeness caused by the informational advantage of institutional investors, or by their participation in delegated investment? How do different forms of delegation contracts affect the risky asset price and price informativeness? To explore these questions, we divide investors with informational advantages into two categories, those involved in delegated investment, and those making direct investment. We examine the influence of the proportion of informed traders participating in delegated investment on price informativeness. In order to analyze the mechanism of institutionalization pushing up the risky asset price—specifically, the impact of different forms of compensation contract on the risky asset price—we categorize institutional investors involved in delegated investment into three groups per their contracts. The first group is non-benchmarked institutions, the second group is benchmarked institutions whose performance is evaluated against a linear benchmark, and the third group is benchmarked institutions with a nonlinear benchmark. This study compares the impacts of these different forms of benchmarking in delegated investments on the risky asset price and price informativeness.

This study offers several important findings that are in sharp contrast to those with the conventional linear benchmarking. First, with the nonlinear benchmark, increasing the penalty intensity can stimulate an institution's information acquisition effort. Under nonlinear benchmarking, increasing the proportion of benchmarked institutions can improve price informativeness and aggregate informational efficiency and reduce the risky asset's return volatility. Second, the nonlinear benchmark, and the resultant nonlinear penalty term, can resolve the sub-optimal problem in information acquisition and price informativeness under conventional performance-based contracts, and mitigate price distortion caused by institutions' pegging to the benchmark portfolio. In all these regards, nonlinear benchmarking is superior to not only linear benchmarking but also non-benchmarking. Compared to the linear penalty term, the effect of the nonlinear penalty term on the institution's utility depends on its penalty intensity. At relatively low penalty intensity, the nonlinear penalty can improve the institution's expected utility and save fixed delegation costs for investors. Third, only raising the average incentive slope can motivate institutions to pursue better information. Changing the incentive slope for an individual institution does not affect its information acquisition. Fourth, compared to direct investment by institutions with informational advantages (i.e., as informed traders), their delegated investment is more conducive to

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[2] Stoughton (1993) and Admati and Pfleiderer (1997) find no motivating effect by the incentive slope on institutional information acquisition effort. Some more recent studies, such as Sheng and Ma (2007), Kyle et al. (2011), and Huang et al. (2020), approach the topic in an endogenous price framework and reveal the incentive slope's motivating effect. The origin for such distinct conclusions is whether changing the incentive slope can affect the equilibrium price. In an effort to unify the two strands of research, this paper takes a new perspective that assumes continuous institutions in a perfectly competitive market and, at the same time, allows variation and change in the incentive slope for an individual institution. This approach enables an investigation of the impact of incentive slope from both individual and group perspectives of institutions. This unified framework avoids the modelling choice between exogenous price and endogenous price that restricts research on the impact of the incentive slope.

improving price informativeness. The greater the proportion of informed traders participating in delegated investment, the higher the price informativeness, and the greater the increase in the risky asset price. Even if informed traders are non-benchmarked investors, their participation in delegated investment per se can improve price informativeness. Different benchmark choices can strengthen or weaken the impact of delegated investment. Compared with the linear benchmark, the nonlinear (quadratic) benchmark can strengthen the contribution of delegated investment to price informativeness and weaken the influence of institutions' excessive holding of the benchmark portfolio on the risky asset price.

The balance of the paper proceeds as follows. Section 2 describes the model framework. Section 3 discusses the issues with linear benchmarked contracts in terms of their impact on information acquisition, investment strategy, equilibrium price, and price informativeness. Section 4 is the core of the paper. First, it introduces a performance-based contract in which the benchmark is a quadratic function of the benchmark portfolio return. Then the impact of this new contract is studied. Comparison with corresponding results under the conventional linear benchmarked contract demonstrates the advantages of the proposed contract in stronger information acquisition incentives, enhanced price informativeness, and reduced price distortion. Finally, effects of the incentive slope under the contract with a quadratic benchmark are investigated. Section 5 investigates the optimal quadratic contract when the principal is risk neutral or risk averse. Section 6 concludes. Proof of the theorems are collected in the appendixes.

2 The model

In this study, our interest is to explore how different forms of benchmarked compensation contracts can affect institutional investors' information acquisition and investment decision-making, as well as the risky asset price and its price informativeness.

Similar to Nezafat et al. (2017) and Breugem and Buss (2019), we consider a three-period model that involves both information acquisition and investment decision-making. In this model, it is assumed that investors (who have money to invest) delegate investment to professional institutions (referred to as institutional investors or institutions hereafter) who possess information advantages and expertise. In period 1, institutions acquire information on the risky asset given their contracts and information acquisition cost. In period 2, institutions construct their portfolios based on the risky asset' payoff information acquired. In period 3, institutions receive their compensation and consume.

2.1 Investment opportunities

Two financial securities are traded in the market: a riskless asset (e.g., a bond) and a risky asset (e.g., a stock market index). The riskless asset is available in perfectly elastic supply. Without loss of generality, the price of the riskless asset is normalized to 1, with the risk-free return 0. The risky asset's price, P, is determined by collective trading (i.e., market clearing) in period 2. Its random payoff X, which follows a normal distribution with mean μ_X and variance σ_X^2, is only observable in period 3. The supply of the risky asset, denoted by Z, is assumed random and unobservable, following a normal distribution as $Z \sim N(\mu_Z, \sigma_Z^2)$. This assumption preserves the incentives to acquire private information by preventing the price from fully revealing the information acquired by investors.

2.2 Institutional investors

There exists a continuum of investors with mass of one. That is, an institutional investor in the market is numbered i, where i follows the uniform distribution on [0, 1]. Institutional investors are separated into two groups: benchmarked institutions (BI) and non-benchmarked institutions (NI). The performance of a benchmarked institution is evaluated against a benchmark, and it will be rewarded (punished) for beating (trailing) the benchmark. Non-benchmarked institutions are not subject to relative performance evaluation. They are remunerated only in proportion to the value of the assets under management (AUM).

Institution i is endowed with the initial wealth under management W_{0i} in period 1. In order to make a sound investment decision, in period 1 institution i acquires a private signal $Y_i = X + \varepsilon_i$ regarding the risky asset's payoff, where the noise term follows $\varepsilon_i \sim N(0, 1/q_i)$ and q_i represents information precision. Higher precision reduces the uncertainty regarding the stock's payoff at an increasing information acquisition cost. Based on the observed payoff information Y_i and price P, in period 2 institution i takes a position θ_i in the risky asset. The conditional expectation of the risky asset payment X is $\widehat{\mu}_{Xi} = E_2(X|Y_i, P)$, and the conditional information precision is $h_i = (Var_2(X|Y_i, P))^{-1}$. The wealth under management in period 3 is then $W_i = W_{0i} + (\theta_i X - \theta_i P)$.

Assume that institutional investors' coefficient of risk aversion is ρ, and their compensation in period 3 is C_i. Following Kyle et al. (2011) and Garleanu and Pedersen (2018), institutions' expected utility satisfies.

$$U_i = -E_1\left(\frac{1}{\rho} ln\big(-E_2\big[-exp(-\rho C_i)|Y_i, P\big]\big)\right). \tag{1}$$

Such a preference indicates that the institution prefers early uncertainty resolution. When C_i is a linear function of X, the utility function (Eq. 1) is essentially the Kreps mean–variance preference.[3] For clarity, we denote the unconditional expectation and variance as $E_1(\cdot)$ and $Var_1(\cdot)$, and the conditional ones as $E_2(\cdot|Y_i, P)$ and $Var_2(\cdot|Y_i, P)$ respectively.

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[3] To demonstrate robustness, the main theorems under the CARA utility function are proved in Appendix B.

2.3 Benchmarked contracts

As in Cuoco and Kaniel (2011) and Buffa et al. (2014), institutional investors' investment performance is evaluated against a benchmark B, which consists of the riskless asset and the risky asset. The weight of the risky asset in the benchmark is β, and the initial level of the benchmark is $W_{0,b}$. Without loss of generality, assume $W_{0,b} = W_{0i}$. In period 3, the level of the benchmark B is $W_B = W_{0,b} + \beta(X - P)$. Since the value of β does not affect the analysis of the impact of contract parameters (γ_i and η_i, as will be introduced below) on the institution's investment behavior, without loss of generality, assume $\beta = 1$. Then the benchmark return is $R_B = \frac{W_B - W_{0,b}}{W_{0,b}} = \frac{X-P}{W_{0i}}$.

The institution's investment performance is $R_P = \frac{W_i - W_{0i}}{W_{0i}} = \theta_i \frac{X-P}{W_{0i}}$. The compensation of the institutional investor in period 3 is

$$\omega_i = W_{0i}[a_{1i} + \eta_{1i} R_P + \gamma_i (R_P - f(R_B))], \tag{2}$$

where $f(R_B)$ is a generalized benchmark, which can be linear in R_B as in conventional benchmarking, or nonlinear in R_B. The institutional investor will be rewarded if its performance beats $f(R_B)$, or be punished if its performance lags $f(R_B)$. Different choice of $f(R_B)$ modifies the compensation contract, causing diverse sensitivity to R_B.[4]

The institutional investor's compensation, Eq. (2), can be rewritten as

$$\omega_i = a_i + \eta_i W_i - \gamma_i W_{0i} f(R_B) \tag{3}$$

where $a_i = W_{0i} a_{1i}$ and $\eta_i = \eta_{1i} + \gamma_i$. Equation (3) show that the institutional investor's compensation has three components: a fixed compensation, a component proportional to AUM, and a penalty term induced by benchmarking. η_i is the incentive slope, representing the incentive strength to the institutional investor. The third component represents the penalty for falling behind the benchmark, where γ_i indicates penalty intensity. Therefore, the compensation contract (Eq. 3) represents the intuitive carrot-and-stick theme.

Chen et al. (2020) believe that a punishment scheme for the agent is necessary when moral hazard exists. Sotes-Paladino and Zapatero (2022) has demonstrated that the penalty term plays an important role in balancing the difference in risk preferences between investors and institutions and ensuring investors' expected return is met. The penalty term $\gamma_i W_{0i} f(R_B)$ is not directly affected by institutions' investment strategy, but it affects their information acquisition and investment decisions. This study explores how an ingenious choice of $f(R_B)$ may enable investors to incentivize institutions to acquire information, thereby improving price informativeness and market efficiency.[5]

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[4] Basak et al. (2008), Cuoco and Kaniel (2011), and Lee et al. (2019) study asymmetric performance-based contracts and find that a nonlinear penalty term can improve institutions' sensitivity to benchmark performance. When $f(R_B)$ is a convex and increasing function of R_B, the better the benchmark's performance, the higher the institutions' sensitivity to benchmark performance.

[5] Increasing the incentive slope η_i to raise the reward, and/or increasing γ_i or modifying $f(R_B)$ to increase penalty for subpar performance, fit the carrot-and-stick motivational approach. Cuoco and Kaniel (2011) and

$\gamma_i \neq 0$ suggests a benchmarked institution (BI), while $\gamma_i = 0$ indicates a non-benchmarked institution (NI). The distinction between the compensation contracts of BI and NI lies in whether the contract involves a benchmark portfolio and thus has a penalty term $\gamma_i W_{0i} f(R_B)$. When accounting for information acquisition cost, the institution's net compensation is

$$C_i = a_i + \eta_i W_i - \gamma_i W_{0i} f(R_B) - k(q_i) \tag{4}$$

where $k(q_i)$ is the information acquisition cost, which satisfies $k'(q_i) > 0$ and $k''(q_i) > 0$.

Assume all institutions have the same initial wealth ($W_{0i} = 1$), then the contract specifics—η_i, γ_i, and $f(R_B)$—and the proportion of benchmarked institutions λ, jointly affect institutions' information acquisition and investment decisions, which in turn impact the price and price informativeness of the risky asset. Section 4 will investigate how a nonlinear benchmark can solve the linear benchmarked contract's suboptimal problems in information acquisition and price informativeness. In Sect. 5, we'll turn our focus to the format of the optimal quadratic contract.

2.4 Investment strategy, information acquisition, and market equilibrium

The optimal strategy of an institutional investor has two components:

1. Optimal portfolio strategy: given its contract and acquired information in period 1, in period 2 institutional investor i chooses exposure to the risky asset, θ_i, to maximize its expected utility:

$$\widehat{\theta_i} = \underset{\theta_i}{argmax}\, E_2[-exp(-\rho C_i)|Y_i, P]. \tag{5}$$

2. Optimal information acquisition: anticipating its optimal investment strategy, $\widehat{\theta_i}$, in period 1 institutional investor i chooses the precision of its private signal, q_i, to maximize its expected utility:

$$\widehat{q_i} = \underset{q_i}{argmax}\, E_1\left[-\frac{1}{\rho} ln\left(-\underset{\theta_i}{max}\, E_2[-exp(-\rho C_i)|Y_i, P]\right)\right]. \tag{6}$$

The rational expectations equilibrium satisfies the following conditions:

(1) The optimal strategies $\widehat{\theta_i}$ and $\widehat{q_i}$ satisfy optimization problems (Eq. 5) and (Eq. 6);
(2) The risky asset's price satisfies the market clearing condition $\int_0^1 \widehat{\theta_i}(P, \widehat{q_i}) di = Z$;
(3) The average precision of private information implied by aggregating investors' precision choices satisfies optimization problems (Eq. 5) and (Eq. 6).

Footnote 5 continued
Huang et al. (2020) study the effect of changing the incentive slope η_i. Breugem and Buss (2019) investigate the impact of adjusting the penalty intensity γ_i. However, there has been scant literature on the effect of modifying the sensitivity of the compensation to benchmark return. This is where this paper contributes to the literature. As will be shown in Sect. 4, when $f(R_B)$ is a convex function of R_B, the contract is more effective in encouraging information acquisition and thus improving price informativeness.

3 Contract with linear benchmark

In Eq. (3), if we set the incentive slope $\eta_i = 1$ (the impact of η_i taking other values will be discussed in Sect. 4.4), the constant $a_i = 0$, the initial wealth $W_{0i} = 1$, and the penalty term $\gamma_i f(R_B) = \gamma_i R_B$, the model is reduced to that in Breugem and Buss (2019), where the compensation contract is

$$\omega_i = W_i - \gamma_i^L W_{0i} R_B = W_{0i}\Big[1 + \Big(1 - \gamma_i^L\Big) R_P + \gamma_i^L (R_P - R_B)\Big] \tag{7}$$

where γ_i^L is the penalty intensity of linear benchmarking. Since Eq. (7) is linear in the payoff of the risky asset, the utility function (Eq. 1) can be transformed into the mean–variance utility in Breugem and Buss (2019).

For non-benchmarked institutions, the compensation contract (Eq. 7) becomes $\widetilde{\omega}_i = W_i = W_{0,i} + \theta_i(X - P)$. For benchmarked institutions, the compensation contract (Eq. 7) is $\widehat{\omega}_i = W_i - \gamma_i^L W_{0i} R_B = W_{0,i} + (\widehat{\theta}_i - \gamma_i^L)(X - P)$.

Given a continuum of institutional investors in the market, an individual institution cannot change the risky asset price. Both benchmarked and non-benchmarked institutions make their investment decisions under a given asset price. When γ_i^L changes, a benchmarked institution can adjust $\widehat{\theta}_i$ to make its risk exposure the same as that of a non-benchmarked institution, that is, $\widehat{\theta}_i - \gamma_i^L = \theta_i$. Then the two institutions have the same compensation, $\widetilde{\omega}_i = \widehat{\omega}_i$, so that the effect of the penalty term ($\gamma_i f(R_B) = \gamma_i R_B$) is completely offset by the benchmarked institution's investment strategy. Since such an adjustment requires no information cost (as γ_i^L is known in the contract), benchmarked institutions have no incentive to acquire more accurate private signal at an increasing cost. Therefore, changing γ_i^L cannot motivate institutions to exert efforts to acquire information.

Summarized below are the main results in Breugem and Buss (2019) regarding investment strategy, equilibrium asset price, information precision, and price informativeness. Under the incentive contract (Eq. 7):

1. The optimal demand for the risky asset is

$$\widehat{\theta}_i^L = \frac{\widehat{\mu}_{Xi} - P^L}{\rho} h_i + \gamma_i^L, \tag{8}$$

   where the mean–variance component, $\frac{\widehat{\mu}_{Xi} - P^L}{\rho} h_i$, is information sensitive but independent of its benchmarking concern (γ_i^L). The hedging demand, γ_i^L, is information insensitive. $\widehat{\theta}_i^L$ is increasing in the hedging demand γ_i^L.
2. The equilibrium risky asset price is

$$P_L = \frac{1}{H_L}\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \overline{q}_L}{\sigma_Z^2 \rho} + \rho \overline{\gamma}_L\right) + \frac{1}{H_L}\left(H_L - \frac{1}{\sigma_X^2}\right) X - \frac{1}{H_L}\left(\frac{\overline{q}_L}{\sigma_Z^2 \rho} + \rho\right) Z, \tag{9}$$

where $H_L = Q_L + \overline{q}_L$, $Q_L = \frac{1}{\sigma_X^2} + \frac{\overline{q}_L^2}{\rho^2\sigma_Z^2}$, $\overline{q}_L = \int\limits_0^{\lambda_L} q_i^{BI} di + \int\limits_{\lambda_L}^{1} q_i^{NI} di$, and $\overline{\gamma}_L = \int\limits_0^{\lambda_L} \gamma_i^L di$. q_i^{BI} and q_i^{NI} represent acquired information accuracy by a benchmarked and a non-benchmarked institution, respectively. $\overline{q}_L$ is the average information accuracy, and $\overline{\gamma}_L$ is the average penalty intensity under the linear contract. Other factors being equal, as more institutions have the performance-based contract as in Eq. (7), the equilibrium asset price will rise. Intuitively, the excess demand stemming from benchmarked investors' hedging drives up the price.

3. An institution with mean–variance utility chooses the optimal information precision $\widehat{q}_i^L$ that satisfies

$$k'\left(\hat{q}_i^L\right) = \frac{1}{2\rho}\left(\frac{1}{H_L^2}\left(\rho^2\left(\sigma_Z^2 + (\mu_Z - \overline{\gamma}_L)^2\right) + H_L + \overline{q}_L\right)\right). \tag{10}$$

   γ_i^L has no incentive effect on an institution's information acquisition. With the mean–variance utility, price informativeness is decreasing in $\overline{\gamma}_L$. Price informativeness is decreasing in λ_L, while the risky asset's return volatility is increasing in λ_L.

Let's analyze why the conventional linear contract (Eq. 7) reduces price informativeness and increases return fluctuation. Under linear benchmarking, the institution's investment strategy is $\widehat{\theta}_i^L = \frac{\widehat{\mu}_{Xi} - P_L}{\rho} h_i + \gamma_i^L$. The hedging demand, γ_i^L, is information insensitive. When the proportion of institutions adopting the linear benchmarked contract, λ_L, rises, the information-insensitive aggregate hedging demand $\int_0^{\lambda_L} \gamma_i^L di$ by benchmarked investors will increase, which reduces aggregate holding of the risky asset in institutions' portfolios $Z - \int_0^{\lambda_L} \gamma_i^L di$ that is sensitive to private information. This makes the informativeness of the risky asset price under contract (Eq. 7) lower than that without benchmarked institutions (i.e., $\lambda_L = 0$). That is, with linear benchmarking, γ_i^L has no incentive effect on an institution's information acquisition. With the mean–variance utility, as the proportion of institutional investors with linear benchmarked contracts (λ_L) rises, the risky asset price increases, price informativeness decrease, and the risky asset's return volatility increases.

Another concern with the performance-based contract (Eq. 7) is that the institution investor will be rewarded even if its investment return is negative, as long as its performance beats the benchmark return ($R_P - R_B > 0$). However, in reality, investors—most of whom are risk averse—loathe negative returns. An intriguing question is, then, whether it is possible for investors to incentivize institutions to strive for positive returns by adjusting the contract's benchmark $f(R_B)$. After all, even in a bear market institutional investors that do great research can still accomplish positive returns by shorting.

Following Basak and Pavlova (2013), the compensation contract (Eq. 7) can be rewritten in terms of wealth as

$$\omega_i = \left(1 - \gamma_i^L\right) W_i + \gamma_i^L (W_i - W_{0i} R_B) \tag{11}$$

As per Eq. (11), an institutional investor will be rewarded even with negative wealth ($W_i < 0$) as long as it beats the benchmark ($W_i - W_{0i} R_B > 0$). This is unacceptable especially to highly risk-averse investors, as investors' utility with negative wealth is often assumed negative infinity (Cuoco and Kaniel 2011; Lioui and Poncet 2013; Escobar-Anel et al. 2020). Therefore, investors have strong interest in modifying the penalty in the contract, requiring positive terminal wealth and punishing contracted institutions in case of negative wealth.

Evidently the linear penalty in the relative-performance contract (Eq. 7) is unable to incentivize institutions to strive to obtain information, resulting in deteriorating market informational efficiency and increasing return fluctuation. Can we design a better contract that can incentivize institutions? In a carrot-and-stick incentive scheme, besides offering a bigger carrot—that is, raising the incentive slope η_i—how about revamping the stick? Can the contract offer rewards only when investment return and wealth are positive?

4 Contract with nonlinear benchmark

In this section, upon proposing the nonlinear benchmarked contract—against a quadratic function of the benchmark return, specifically—we first explore the optimal investment strategy, equilibrium price, and information precision under the novel contract. Then we demonstrate the advantages of quadratic benchmarking over the conventional linear benchmarking in terms of incentive of information acquisition, price informativeness, return volatility, and institutions' utility. Last, we investigate the impact of the incentive slope on information acquisition and price informativeness.

4.1 Performance-based contract with quadratic benchmark and penalty

Due to the failure of performance-based contract with the conventional linear benchmark and penalty as discussed in Sect. 3, the penalty term in the contract, $\gamma_i W_{0i} f(R_B)$, should meet the following conditions.

1. $\gamma_i W_{0i} f(R_B)$ should contain no simple hedging term that would enable the institution to offset benchmarking risk by adjusting its risk exposure at no information cost. It is important to maintain information sensitivity in the demand for the risky asset.
2. The specification of $\gamma_i W_{0i} f(R_B)$ should make its sensitivity to the risky asset's return ($X - P$) higher than the sensitivity in the absence of benchmarking. The heightened sensitivity would motivate institutions to exert additional effort to improve the precision of private information, in their pursuit of higher compensation.
3. The contract with the new benchmark and penalty term should be acceptable to some institutional investors (so that $\lambda \neq 0$). Specifically, the following two conditions are met: (i) the institution's participation constraint is satisfied; (ii) when adopting the optimal information acquisition strategy and investment strategy, the institution's expected utility is not lower than that under the non-benchmarked

contract $\omega_i = a_i + \eta_i W_i$ or the conventional benchmarked contract with $f(R_B) = R_B$.

4. Since investors loathe negative returns, the contract should reward institutions only if their investment return is positive and beats the benchmark. Otherwise, institutions should be penalized.

Based on these considerations, the generalized benchmark $f(R_B)$ should have the following properties:

1. $f(R_B)$ is increasing in R_B when $R_B \geq 0$;
2. $f(R_B) \geq 0$;
3. $f(R_B) = f(X - P)$ shall encourage the pursuit of greater information precision by elevating its sensitivity to return $X - P$.

In the light of the foregoing along with model trackability, with reference to Stoughton (1993) and Cvitanić and Xing (2018), let

$$f(R_B) = \frac{1}{h}(R_B)^2 \geq 0. \tag{12}$$

$f(R_B)$ is a convex function of R_B, ensuring that the higher the benchmark return R_B, the higher the sensitivity of the institution's compensation to benchmark performance. To distinguish from the linear benchmarked contract, denote $f(R_B)$ in Eq. (7) as $f_L(R_B) = R_B$. $h > 0$ is a threshold chosen by investors. The specification of $f(R_B)$ in Eq. (12) has the following characteristics.

- When $R_B \geq 0$, if $h > R_B$, $f(R_B) = (R_B)^2/h < R_B$. The benchmark is lower than the conventional benchmark. A lower benchmark (and thus lower penalty) suggests that investors are more concerned about the sign (positive or negative) of the rate of return. Investors lower the benchmark and penalty in order to encourage institutions to always obtain positive investment return.
- In favorable market conditions, a lower threshold is chosen ($h \leq R_B$) so that $f(R_B) = (R_B)^2/h \geq R_B$. The higher benchmark signifies investors' high expectations on institutions to deliver satisfactory returns in advantageous market conditions.
- The symmetric functional form $f(R_B) = (R_B)^2/h$ indicates that even if the benchmark return is negative, investors still want institutions to maintain the same sensitivity as when the benchmark return is positive. That is, investors expect institutions to be able to predict unfavorable market conditions ($R_B < 0$) in advance by acquiring accurate information so that they can earn positive returns by short selling the risky asset.

With Eq. (12) , the performance-based contract (Eq. 2) can be rewritten as

$$\omega_i = W_{0i}[a_{1i} + \eta_{1i} R_P + \gamma_i (R_P - (R_B)^2/h)], \tag{13}$$

and the contract (Eq. 3) in terms of the terminal wealth can be expressed as

$$\omega_i = a_i + \eta_i W_i - \gamma_i W_{0i}(R_B)^2/h = a_i + \eta_i W_{0i} + \left[\eta_i\theta_i - \gamma_i(X-P)/(W_{0i}\mathrm{h})\right](X-P). \tag{14}$$

Equations (13) and (14) are equivalent. Since the benchmark $f(R_B) = (R_B)^2/h \geq 0$, the institution is rewarded only if its investment return is positive and higher than the benchmark. Otherwise, it will be subject to penalty.

As per Eq. (14) , in order to earn a reward, when $X - P > 0$, the institution should invest in the risky asset to make $\eta_i\theta_i - \gamma_i(X-P)/(W_{0i}h) > 0$. In contrast, when $X - P < 0$, it should short sell the risky asset to make $\eta_i\theta_i - \gamma_i(X-P)/(W_{0i}h) < 0$. The key is, then, whether the institutional investor can correctly predict the payoff of the risky asset and make the right investment decision (long or short) accordingly. Failure to do so will subject itself to a reduced compensation. Such a contract demands institutions to invest in information acquisition. To simplify analytical results, $h = 1$ and $W_{0i} = 1$ are assumed henceforward without loss of generality.

If $f(R_B) = (X-P)^2$ is regarded as a quadratic option, then institutions, as the party with informational advantage, hold the short position of the option, and the investors (principals) are in the long position. Brennan and Cao (1996) believe that the allocation involving the quadratic option is Pareto efficient. Different from Brennan and Cao (1996), $f(R_B) = (R_B)^2/h$ serves as the benchmark in delegation contracts in this study, which reflects the principal's required investment performance. That is, investors expect the institutions to earn a positive rate of return and bear the tail risk and penalties brought by subpar performance. Investors specify their expected return in the form of benchmark in delegation contracts to incentivize institutions to pursue information and active investment.

The quadratic penalty term reflects investors' demand for institutions to elevate information acquisition efforts to take full advantage of their information advantage. Given the institution's information acquisition and the market price, the expected loss caused by the penalty term is:

$$\begin{aligned} E[(\gamma_i W_{0i}(R_B)^2/h)|Y_i, P] &= \gamma_i E[(X-P)^2|Y_i, P] \\ &= \gamma_i\left[E\left(X^2|Y_i, P\right) - 2PE(X|Y_i, P) + P^2\right] \\ &= \gamma_i[Var(X|Y_i, P) + (E(X|Y_i, P))^2 - 2PE(X|Y_i, P) + P^2] \\ &= \gamma_i Var(X|Y_i, P) + \gamma_i(E[X|Y_i, P] - P)^2 \end{aligned}$$

The higher the observed accuracy $(Var(X|Y_i, P))^{-1}$ by an institutional investor of the risky asset's payoff, the smaller $Var(X|Y_i, P)$, and thus the lower the penalty. Therefore, the quadratic benchmark penalizes the conditional variance, which incentivizes portfolio managers to acquire more precise signal.[6]

There are several additional points to note regarding Eq. (14) :

1. It is assumed that the fixed compensation a_i, along with the incentive slope η_i, is sufficient to cover the information acquisition cost, so that the institution's participation constraint is always satisfied.

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[6] We appreciate an anonymous reviewer suggesting this interpretation of the quadratic penalty term.

2. As discussed with regard to Eq. (7), when the penalty intensity γ_i^L increases, the institution can always adjust its risk exposure to maintain $\widehat{\theta}_i - \gamma_i^L$ unchanged at no information cost, so that the benchmarking effect is neutralized. In contrast, under the contract as in Eq. (14) , $\eta_i \theta_i - \gamma_i(X - P)$ is contingent on $X - P$, which is unknown prior to period 3. Therefore, there is no straightforward way to offset the benchmarking effect. The best an institution can do is to acquire more precise private signal regarding $X - P$ in order to devise an appropriate investment strategy.
3. $f(R_B) = (X - P)^2$ is unaffected by an individual institution's investment choice, but it affects its information gathering and investment strategy. The penalty term $\gamma_i(X - P)^2$ makes the compensation of benchmarked institutions more sensitive to the risky asset's return than that of non-benchmarked institutions. A high penalty intensity γ_i would push the institution to work hard in information gathering in order to minimize or avoid the penalty.

The exploration of the impact of nonlinear benchmarking extends Stoughton (1993) and that study the effects of delegation contracts on asset prices and price informativeness. As shown below, quadratic benchmarking can motivate institutional investors to acquire information, improve market price informativeness, and reduce fluctuation in returns. Such contracts can better serve investors, especially those with a high level of risk aversion.

4.2 Information acquisition, equilibrium price, and price informativeness

For convenience in reference, the conventional linear benchmarking in Eq. (7) is labelled as LB, while the quadratic benchmarking in Eq. (13) is labelled as QB. This section derives the analytical results for the optimal strategy of the institution, $\widehat{\theta}_i$ and $\widehat{q}_i$, and the equilibrium price P, under a QB contract.

4.2.1 Optimal investment strategy and equilibrium price

Theorem 1 *Given the private information precisionq_i and the risky asset price P, the institution's optimal exposure to the risky asset is*

$$\widehat{\theta}_i(q_i) = \frac{(\widehat{\mu}_{Xi} - P)}{\rho \eta_i} h_i. \tag{15}$$

Proofs of all theorems are collected in Appendix A. under an LB contract, the institution's optimal risk exposure $\widehat{\theta}_i^L = \frac{\widehat{\mu}_{Xi} - P_L}{\rho} h_i + \gamma_i^L$ includes a hedging component γ_i^L that is information insensitive. In contrast, under a QB contract the optimal risk exposure does not explicitly involve penalty intensity γ_i. However, penalty intensity may affect the institution's optimal exposure to the risky asset via its acquired information accuracy q_i. To formally investigate the impact of γ_i on $\widehat{\theta}_i(q_i)$, we need to solve information accuracy q_i and examine the impact of γ_i on q_i. Under a QB contract the optimal risk exposure is similar to that of non-benchmarked institutions, i.e., as *the result of a mean–variance optimization. Since* $\widehat{\theta}_i(q_i)$*is determined by* $\widehat{\mu}_{Xi}$*and* h_i*-both are information sensitive—an institution must invest in information acquisition to make a sound investment decision. Thus, non-linear benchmarking in the* QB *contract does not lead to a decrease in information-sensitive asset demand.*[7]

Theorem 2 *There exists a unique linear rational expectations equilibrium in which the risky asset price satisfies*

$$P = \left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \overline{q}}{\sigma_Z^2 \rho \overline{\eta}}\right) H^{-1} + H^{-1}\left(H - \frac{1}{\sigma_X^2}\right) X - H^{-1}\left(\frac{\overline{q}}{\sigma_Z^2 \rho \overline{\eta}} + \rho \overline{\eta}\right) Z, \quad (16)$$

where $H = Q + \overline{q}$, $Q = \frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2 \overline{\eta}^2 \sigma_Z^2}$, $\overline{q} = \int_0^{\lambda_Q} q_i^{BI} di + \int_{\lambda_Q}^{1} q_i^{NI} di$, $\overline{\eta} = \left(\int_0^1 \frac{1}{\eta_i} di\right)^{-1}$,

and $\overline{\gamma}_Q = \int_0^{\lambda_Q} \gamma_i di$.

In Eq. (16), only $\overline{\eta}$ —the average incentive slope—affects the equilibrium price, but individual η_i does not. λ_Q represents the proportion of institutions adopting the QB contract, $\overline{\gamma}_Q$ represents the average penalty intensity, and $\overline{q}$ represents the average precision of acquired information. The economic meaning of Q and H will be discussed in Sect. 4.2.3. Following Huang et al. (2020), we call $\rho\overline{\eta}$ the effective risk aversion.

The equilibrium price under the *QB* contract, Eq. (16), is consistent with the equilibrium price when the market is populated only with non-benchmarked institutions. However, due to the nonlinear penalty term $\gamma_i (R_B)^2$, information acquisition and investment decisions by institutions under the *QB* contract significantly differ from those by non-benchmarked institutions. Therefore, to study the impact of the *QB* contract on equilibrium asset price and price informativeness, it is necessary to solve the optimal information precision first.

4.2.2 Optimal information precision

The optimal precision in information acquisition maximizes the expected utility of the institution. It is a tradeoff between two considerations. On one hand, more precise information improves the quality of the institution's investment strategy, so that a higher expected return can be earned. On the other hand, higher information precision comes at an increasing information acquisition cost, which reduces the expected utility. Plug $\widehat{\theta}_i$ into U_i and let

$$\widehat{q}_i = \underset{q_i}{argmax} E_1\left[-\frac{1}{\rho} ln\left(-\underset{\theta_i}{max} E_2(-exp(-\rho C_i)|Y_i, P)\right)\right]. \quad (17)$$

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[7] In contrast, under LB contracts, the hedging demand is insensitive to information. As more institutions adopt such contracts, the proportion of information-sensitive demand for the risky asset would decline.

Theorem 3 *Given the average information precision, $\overline{q}$, investor i's optimal information precision $\hat{q}_i(\overline{q})$satisfies the following equation*:

$$k'(\hat{q}_i) = \frac{1}{2\rho}\left(\frac{1}{Q+\hat{q}_i-2\rho\gamma_i} - \frac{1}{Q+\hat{q}_i} + H^{-2}\left(\rho^2\overline{\eta}^2\left(\sigma_Z^2+\mu_Z^2\right)+H+\overline{q}\right)\right). \quad (18)$$

The optimal information precision $k'\left(\hat{q}_i^L\right) = \frac{1}{2\rho}\left(\frac{1}{H_L^2}(\rho^2\left(\sigma_Z^2 + \left(\mu_Z - \overline{\gamma}_L\right)^2\right) + H_L + \overline{q}_L)\right)$ *under the* LB contract (Breugem and Buss 2019), *does not involve* γ_i^L. In contrast, under the QB contract, the optimal information precision, $\widehat{q_i}$ explicitly entails γ_i. This is because under the LB contract, any loss caused by the linear penalty term can be costlessly hedged (as discussed in Sect. 3); changing the penalty intensity cannot motivate the institution to obtain better information. Under the QB contract, in contrast, the loss caused by the penalty $\gamma_i(R_B)^2$ cannot be offset by hedging. In order to mitigate the penalty and thus improve compensation, the institution has to improve its information precision. Thus, under the QB contract, γ_i can incentivize information acquisition.

4.2.3 Informativeness of price

Institutions' information acquisition affects price informativeness of the risky asset, which measures its price's ability to convey its payoff information. Kyle et al. (2011), Goldstein and Yang (2019), and Huang et al. (2020) measure price informativeness by the accuracy of the risky asset's payoff information embedded in the price P, $(Var_2(X|P))^{-1}$. The greater the accuracy of the payoff information embedded in P, the higher the price's informativeness. Without loss of generality, we quantify price informativeness as $Q = (Var_2(X|P))^{-1} = \frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}$, which is the result per the Bayes' rule.[8] Price informativeness is public information.

Institutions not only derive information from the risky asset's price but also scout information. Upon acquiring their private signal regarding the payoff information of the risky asset, institutions formulate their investment strategies. Their collective private information is reflected in the price via aggregate demand for the risky asset. The better the information precision obtained by an institution, $h_i = (Var_2(X|Y_i, P))^{-1} = Q + q_i$, the higher the validity of its information. Therefore, we call h_i the informational efficiency obtained by an individual institution, and $H = Q + \overline{q}$ the aggregate informational efficiency. H reflects the market's ability to gather information, where

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[8] Breugem and Buss (2019) define the precision of the public price signal $\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}$ as price informativeness. Because σ_X^2 does not affect the analysis of compensation contracts' impact on price informativeness, $\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}$ and $\frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}$ are equivalent for the purpose of analyzing its fluctuation and monotonicity.

Q is public information obtained from the price, and $\overline{q}$ represents the average precision of private information acquired by institutions. The larger H is, the stronger the ability of the market to gather and transmit information, and the higher the market informational efficiency.

Equation (18) show that information precision is affected by penalty intensity γ_i and average incentive slope $\overline{\eta}$. Next, we will study how γ_i, η_i, and λ_Q affect institutions' information acquisition and investment, which in turn would determine the risky asset's price and its price informativeness.

4.3 Impact of quadratic benchmark

This section discusses the impact of the *QB* contract on institutional information acquisition, price informativeness, and risky asset price. Comparisons to corresponding results under the *LB* contract demonstrate the *QB* contract's advantages in terms of stronger incentive for information acquisition, improved price informativeness, reduced return volatility, and, under low penalty intensity, elevated expected utility of institutions.

4.3.1 Incentive for information acquisition

Since the incentive slope η_i has not impact on how γ_i affects information acquisition, $\eta_i = 1$ is assumed in this section without loss of generality. Denote the information precision by a non-benchmarked institution as $\widehat{q}_i^N$.

Theorem 4

1. *Given* P, $\overline{q}$, Q, σ_Z^2, *and* μ_Z, *the optimal information precision increases with the penalty intensity, i.e.,* $\partial\widehat{q}_i/\partial\gamma_i > 0$. *As long as* $\gamma_i \neq 0$, $\widehat{q}_i > \widehat{q}_i^L$, $\widehat{q}_i > \widehat{q}_i^N$, $\frac{\partial(\widehat{q}_i-\widehat{q}_i^L)}{\partial\gamma_i} > 0$, *and* $\frac{\partial(\widehat{q}_i-\widehat{q}_i^N)}{\partial\gamma_i} > 0$.
2. *Average information precision increases with average penalty intensity, i.e.,* $\partial\overline{q}/\partial\overline{\gamma}_Q > 0$. *An institution's information acquisition effort is affected by both* $\overline{\gamma}_Q$ *and* γ_i. *With an increasing* $\overline{\gamma}_Q$, *when* $\partial\gamma_i/\partial\overline{\gamma}_Q > 0$, $\partial\hat{q}_i/\partial\overline{\gamma}_Q > 0$*; when* $\partial\gamma_i/\partial\overline{\gamma}_Q \leq 0$, *which includes the case of a non-benchmarked institution,* $\partial\hat{q}_i/\partial\overline{\gamma}_Q < 0$.

Theorem 4 shows that under the *QB* contract, raising the penalty intensity can motivate institutions to acquire better information. This is in sharp contrast to the lack of motivating effect under the *LB* contract. Other conditions being equal, the optimal information precision under the *QB* contract, $\widehat{q}_i$, is higher than $\widehat{q}_i^L$ under the *LB* contract, and $\widehat{q}_i^N$ for a non-benchmarked institution. The stronger γ_i is, the greater the margin by which $\widehat{q}_i$ exceeds $\widehat{q}_i^L$ or $\widehat{q}_i^N$.

The non-benchmarked institutions' compensation is $\omega_i = a_i + W_i = a_i + 1 + \theta_i(X - P)$, while the *QB* benchmarked institution's compensation is $\omega_i = a_i + W_i - \gamma_i W_{0i}(R_B)^2$. The convexity of the penalty term $\gamma_i W_{0i}(R_B)^2 = \gamma_i(X - P)^2$ makes the sensitivity to $X - P$ higher for a *QB* benchmarked institution than for a non-benchmarked institution. It is this elevated sensitivity that enables the *QB* contract to resolve the suboptimal information acquisition problem suffered by linear-benchmarked contracts compared to non-benchmarked contracts.

Unlike the case of *LB* contract, the penalty term $\gamma_i W_{0i}(R_B)^2 = \gamma_i (X - P)^2$ in the *QB* contract cannot be easily hedged away. Any misjudgment regarding the sign of $X - P$ would depress the compensation under the *QB* contract. Thus, institutions with the *QB* contract are more motivated than non-benchmarked institutions to improve information precision in order to reduce the chance of misjudgment.

Two contrary forces are at work that affect an institution's information acquisition effort: the direct effect of its penalty intensity, γ_i, and the marketwide effect of average penalty intensity, $\overline{\gamma}_Q$. On one hand, because a larger γ_i would cause a higher loss $\gamma_i W_{0i}(R_B)^2$ to its compensation, the institution is motivated to obtain more precise information to improve investment performance. On the other hand, when average penalty intensity $\overline{\gamma}_Q$ increases, it improves average information accuracy $\overline{q}$ under the *QB* contract. A higher $\overline{\gamma}_Q$ disincentivizes information acquisition effort. This is because the heightened $\overline{q}$, caused by higher $\overline{\gamma}_Q$, makes institutions invest less in information acquisition as they can benefit from increased price informativeness. The net effect then depends on whether the two effects align, or, in case of contrary, which effect dominates. As $\overline{\gamma}_Q$ rises, for a *QB* benchmarked institution that increases γ_i, the direct effect of γ_i dominates the effect of $\overline{\gamma}_Q$ so that information precision enhances ($\partial \hat{q}_i / \partial \overline{\gamma}_Q > 0$); for a non-benchmarked institution or a *QB* benchmarked institution that lowers γ_i, both effects align so that $\partial \hat{q}_i / \partial \overline{\gamma}_Q < 0$.

Breugem and Buss (2019) show that the optimal information precision for non-benchmarked institutions is higher than that for institutions with a linear benchmarked contract. That is, $\hat{q}_i^N \geq \hat{q}_i^L$. As shown above, the information precision is higher for *QB* benchmarked institutions than non-benchmarked institutions, i.e., $\hat{q}_i > \hat{q}_i^N$. Therefore, $\hat{q}_i > \hat{q}_i^L$. The larger γ_i is, the bigger the difference $\hat{q}_i - \hat{q}_i^L$, as shown in Fig. 1, where $\gamma_i = \gamma$. The parameters in all the figures follow Breugem and Buss (2019). Specifically, $\mu_X = 1.05, \sigma_X^2 = 0.25, \mu_Z = 1, \sigma_Z^2 = 0.2, \rho = 3, k(q_i) = c_i q_i^2$, and $c_i = 0.03$. Figure 1 also illustrates that as the penalty intensity increases, the

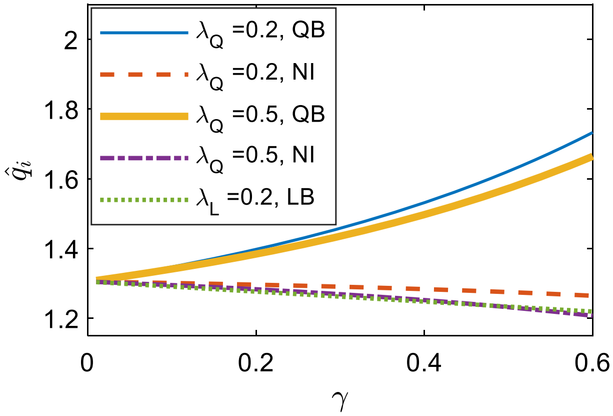

Fig. 1 Impact of γ on $\widehat{q}_i$

information precision for *NI* is reduced to a lesser extent compared to the improvement in information precision under the *QB* contract. The average information accuracy thus increases.

Moreover, as more institutions adopt *QB* contracts (e.g., λ_Q increases from 0.2 to 0.5), the information precision acquired by a *QB* benchmarked institution is lowered. This is because a larger λ_Q elevates average penalty intensity $\overline{\gamma}_Q$, which markedly improves average information precision ($\overline{q}$). The institution thus relies more on the market price for information and invests less in acquiring private information.

4.3.2 Price informativeness

Theorem 4 shows that increasing penalty intensity γ_i in the QB contract can motivate the institution to acquire more precise information. As average penalty intensity $\overline{\gamma}_Q$ elevates, average information precision $\overline{q}$ rises, so will price informativeness *Q* and aggregate informational efficiency H.

To facilitate analysis, assume for now $\gamma_i = \gamma$ as a constant, so $\overline{\gamma}_Q = \gamma\lambda_Q$. Thus, an increase in either the penalty intensity (γ) or the proportion of institutions adopting the *QB* contract (λ_Q), will cause the average penalty intensity $\overline{\gamma}_Q = \gamma\lambda_Q$ to rise, hoisting price informativeness. Denote the price informativeness in the absence of benchmarked contracts as Q_N, and average information precision as $\overline{q}_N$. Denote the price informativeness under the *LB* contract as Q_L, and aggregate informational efficiency as H_L.

Corollary 1 *If no institution in the market has a benchmarked contract,* $Q_N = Q_L = Q$. *When all benchmarked investors in the market adopt the QB contract and* $\lambda_Q \neq 0$, $Q > Q_N$, $\frac{dQ}{d\overline{\gamma}_Q} > 0$, $\frac{d(Q-Q_N)}{d\overline{\gamma}_Q} > 0$, *and* $\frac{\partial H}{\partial \overline{\gamma}_Q} > 0$. *When all benchmarked investors adopt the LB contract and* $\lambda_L \neq 0$, $Q_L < Q_N$, $\frac{dQ_L}{d\overline{\gamma}_L} < 0$, $\frac{d(Q_L-Q_N)}{d\overline{\gamma}_L} < 0$, *and* $\frac{\partial H_L}{\partial \overline{\gamma}_L} < 0$.

Compared to the case without performance-based contracts, the *QB* contract improves price informativeness, while the *LB* contract diminishes price informativeness. As the proportion of institutions adopting the Q*B* (*LB*) contract—λ_Q (λ_L)—rises, the margin in price informativeness $Q - Q_N$ ($Q_N - Q_L$) widens. Nonlinear benchmarking thus solves the conundrum that the conventional linear-benchmarked contracts undermine price informativeness.

The positive impact of λ_Q on price informativeness is shown in Fig. 2, contrary to the case with the *LB* contract. If the penalty intensity is low (e.g., $\gamma = 0.1$), price informativeness increases slowly with λ_Q, dragged by the reduced information precision by non-benchmarked institutions. As a reference for comparison, denote the aggregate informational efficiency in the absence of performance-based contracts as H_N, then $H_L \leq H_N < H$. Among the three contracting situations, the market informational efficiency is the lowest under the *LB* contract and the highest under the *QB* contract. Other things being equal, as more institutions adopt the *QB* (*LB*) contract, the aggregate informational efficiency improves (declines). Linear benchmarking undermines market informational efficiency, while quadratic benchmarking improves it.

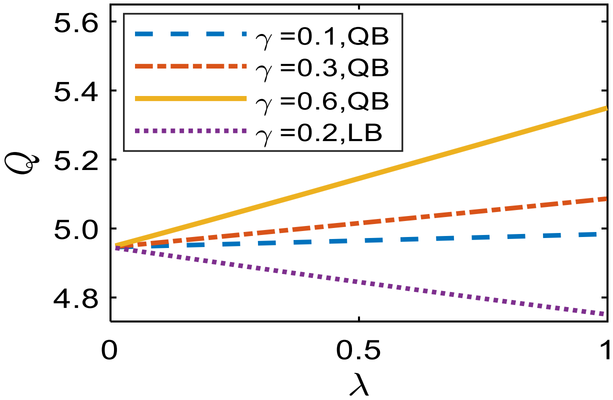

Fig. 2 Impact of λ on price informativeness

4.3.3 Expected price and return volatility

When penalty intensity γ_i increases, the institution acquires more precise private information $\hat{q}_i$, which reduces the uncertainty regarding the payoff of the risky asset. The demand for the risky asset then increases as the institution pursues greater utility. At the market level, when $\overline{\gamma}_Q$ rises, the aggregate demand for the risky asset elevates, resulting in a higher price.

Theorem 5 *Under the QB contract, the demand for the risky asset, its expected price and return satisfy the following conditions*:

$$\frac{\partial E\left(\hat{\theta}_i\right)}{\partial \gamma_i} > 0, \ \frac{\partial E(P)}{\partial \overline{\gamma}_Q} > 0, \ \frac{\partial E(X-P)}{\partial \overline{\gamma}_Q} < 0, \ \text{and} \ \frac{\partial Var(X-P)}{\partial \overline{\gamma}_Q} < 0.$$

When $\overline{\gamma}_Q$ increases, price informativeness Q and aggregate informational efficiency H improve. When institutions as a group have a better grasp regarding the payoff of the risky asset, the perceived investment risk is reduced. At a given level of risk tolerance, stronger aggregate demand boosts the expected price, cutting the expected return.

The risky asset's expected price is positively related to average penalty intensity, whether the benchmarking is linear or nonlinear. The *QB* contract has a smaller impact on the price than the *LB* contract, as shown in Fig. 3. Intuitively, under the *LB* contract, the information-insensitive hedging component in institutions' investment strategy elevates the demand for the risky asset and consequently its price. In contrast, under the *QB* contract, an institution cannot easily hedge its benchmarking risk by adjusting its exposure to the benchmark portfolio. As shown in Eq. (15), its optimal investment strategy contains only the mean–variance investment portfolio. The demand for the risky asset comes at additional informational costs (because $\hat{q}_i > \hat{q}_i^L$). Therefore, under the *QB* contract, the institution's investment strategy is relatively conservative, and the price grows less compared to the comparable situation with the *LB* contract.

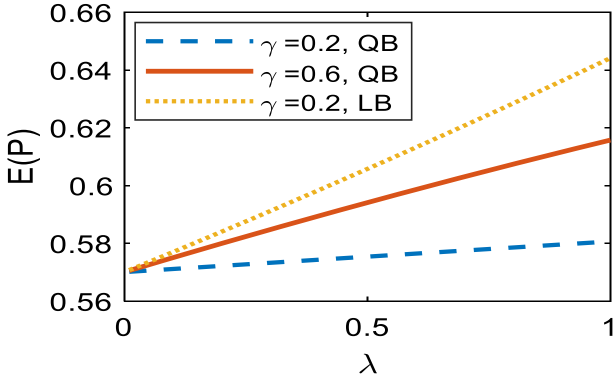

Fig. 3 Impact of λ on expected price

Nonlinear benchmarking can thereby mitigate price inflation caused by institutions' pegging to the benchmark portfolio under linear benchmarking.

For a non-benchmarked institution, the penalty intensity $\gamma_i = 0$; when the market is populated only with non-benchmarked institutions, $\overline{\gamma}_Q = 0$. The expression of risky asset price under the *QB* contract in Eq. (16) is consistent with that when there are only non-benchmarked investors in the market. However, due to the quadratic penalty term, other things being equal, the equilibrium asset price under the *QB* contract is higher than that in a market with only non-benchmarked institutions.

When $\overline{\gamma}_Q$ increases, price informativeness and aggregate informational efficiency improve. As institutions acquire better information regarding the risky asset's prospect, their investment becomes more information driven, which reduces return volatility partially caused by capricious decision making. Therefore, the fluctuation of the risky asset's return is a decreasing function of $\overline{\gamma}_Q$. Denote the return variance under the *QB* contract as $V_Q = Var(X - P)$, under the *LB* contract as $V_L = Var(X - P_L)$, and in the absence of benchmarked contracts as V_N. Then $V_Q < V_N < V_L$, $\frac{d(V_L - V_N)}{d\overline{\gamma}_L} = \frac{dV_L}{d\overline{\gamma}_L} > 0$, and $\frac{d(V_Q - V_N)}{d\overline{\gamma}_Q} = \frac{dV_Q}{d\overline{\gamma}_Q} < 0$. Compared to the case without performance-based contracts, the *LB* contract elevates the return volatility, while the *QB* contract diminishes the volatility. As shown in Fig. 4, as the proportion of institutions adopting the *QB* (*LB*) contract (i.e., λ_Q (λ_L)) increases, the return volatility keeps decreasing (increasing).

4.3.4 Institutions' utility and fixed delegation cost

The analyses thus far have shown that the *QB* contract is superior to the *LB* contract in terms of motivating information acquisition, improving price informativeness and information efficiency, and reducing the inflation of equilibrium price and the return volatility. Is the superiority of the *QB* contract at the expense of higher fixed compensation for incentivizing institutions and improving price informativeness? Or, when an institution obtains equal utility under the two benchmarked contracts, does the initial endowment have to be higher under the *QB* contract?

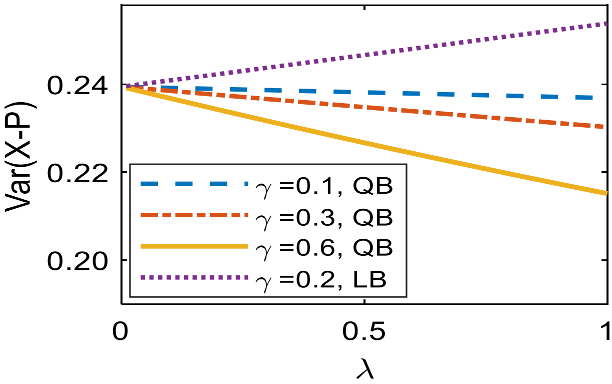

Fig. 4 Impact of λ on return volatility

This section carries out analysis from two perspectives. First, assume that an institution obtains equal utility under *QB* and *LB* contracts, and the initial endowment W_{0i} is the same. Assuming that the fixed income component under the *LB* contract is 0, we investigate whether the fixed income in the *QB* contract is positive or negative. Second, assuming the fixed income components in *LB* and *QB* contracts are both zero and the institution obtains equal utility, we compare the required initial endowment under both contracts.

Assume the fixed income in the *LB* contract is zero, and in the *QB* contract is a. Under the *LB* contract, the institution's utility satisfies

$$U_i^L = W_{0,i} - k\left(\hat{q}_i^L\right) + \frac{h_i}{2\rho}\left(\rho^2\left(\sigma_Z^2 + \left(\mu_Z - \overline{\gamma}_L\right)^2\right) + Q_L + 2\overline{q}_L\right)H_L^{-2} - \frac{1}{2\rho}. \quad (19)$$

Under the *QB* contract, the institution's utility satisfies

$$\begin{aligned} U_i &= a + W_{0,i}\left(1+\xi\right) - k\left(\hat{q}_i\right) + \frac{1}{2\rho}\ln\left|\frac{h_i - 2\rho\gamma_i W_{0,i}\left(1+\xi\right)}{h_i}\right| \\ &\quad + \frac{h_i}{2\rho}\left(\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + Q + 2\overline{q}\right)H^{-2} - \frac{1}{2\rho} \end{aligned} \quad (20)$$

Substitute the information precision, $\widehat{q}_i^L$ and $\widehat{q}_i$, into Eqs. (19) and (20) respectively. Assume equal initial endowment under *LB* and *QB* contracts and zero fixed compensation in the *LB* contract. Based on $U_i^L = U_i$, we solve for the fixed income component (a) in the *QB* contract. The variation of a with γ is shown in Fig. 5, whose parameters are the same as in Fig. 1, and $\gamma_L = 0.3$, $\lambda_L = 0.6$. Figure 5 show that in order to offer the same utility as the *LB* contract, the fixed income in the *QB* contract is contingent on the penalty intensity γ. When γ is in its lower range (approximately less than 0.3), a is negative; otherwise, a is positive. That is, at relatively low penalty intensity, if the fixed income component is the same between the *LB* and *QB* contracts, the institution will gain higher utility under the *QB* contract.

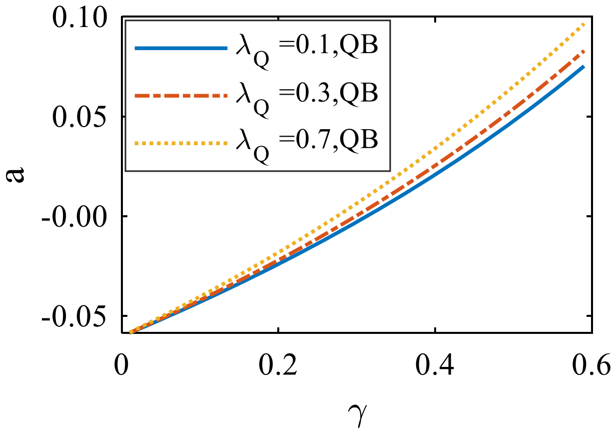

Fig. 5 Impact of γ on a

To compare from another perspective, assume the fixed income is zero in both *LB* and *QB* contracts. Assuming the institution gains the same utility under the two contracts, we compare the initial endowment W_{0i}. To simplify the comparison, the initial endowment under the *LB* contract is standardized to 1. When $U_i^L = U_i$, denote the initial endowment under the *QB* contract as $1 + \xi$. Thus ξ reflects the difference in proportion in endowment for the two contracts to deliver the same utility. Figure 6 depicts how ξ varies with γ, with the same parameters as in Fig. 5.

Figures 5 and 6 illustrate that whether the *QB* contract requires higher fixed compensation or endowment than the *LB* contract to deliver the same utility depends on the level of penalty intensity in the *QB* contract. Under the linear benchmark, an institution's individual penalty intensity γ_i^L does not affect its expected utility $\hat{U}_i^L$, due to hedging in its investment strategy; only average linear penalty intensity $\overline{\gamma}_L$ affects $\hat{U}_i^L$. The information-insensitive hedging leads to a deterioration in price informativeness.

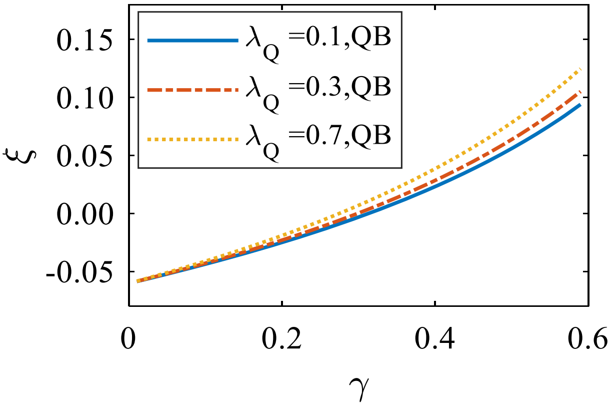

Fig. 6 Impact of γ on ξ

As the price—a valuable source of information—contains more noise, the institution has to exert more effort to acquire the same amount of total information. The cost of information acquisition thus increases. Under the nonlinear benchmark, in contrast, the institution's expected utility is jointly affected by its individual penalty intensity γ_i and average penalty intensity $\overline{\gamma}_Q$. When γ_i is relatively small, the penalty and the cost of information acquisition are low. Compared to the case with *LB* contract, higher information accuracy under the *QB* contract improves the institution's investment performance and thereby increases its end-of-period shared revenue $\eta_i W_i$. Moreover, the improved price informativeness under the *QB* contract helps lower the institution's information acquisition cost, which contributes positively to its utility.

As γ_i rises, to mitigate the potentially higher penalty, the institution has to invest more in information acquisition. As the function of information acquisition cost is convex, it grows faster than information accuracy. The increased revenue $\eta_i W_i$ can no longer compensate for the elevated penalty and information acquisition cost, resulting in lower utility of the institution. To obtain the same utility as under the *LB* contract, the institution would require a higher fixed compensation (a) or a larger endowment (ξ), as shown in Figs. 5 and 6.

Interestingly, as more institutions adopt the *QB* contract, the institution would also require a higher fixed income or a larger pool of funds. This is because as λ_Q increases, more institutions acquire information of better accuracy. Quality investment opportunities in the market are thus tapped by more institutions, resulting in decreasing returns. Therefore, the institution requires a higher fixed fee or more assets to manage to obtain the same expected utility. In Figs. 5 and 6, both a and ξ are increasing in λ_Q.

In summary, compared to the linear benchmarked compensation contract, the quadratic benchmarked contract can incentivize information acquisition, improve price informativeness and aggregate informational efficiency, reduce the impact of institutions' pegging to benchmark portfolio on the price, lower return volatility, and, when penalty intensity is relatively low, reduce investors' fixed delegation cost and enhance institutions' expected utility.

The exploration of nonlinear benchmarked contracts proposes a novel perspective for solving the suboptimal problem of conventional performance-based contracts in information acquisition and price informativeness. It is important to keep in mind that our model assumes no restrictions on short selling. When restrictions on short selling exist, either institutionally or practically, even if institutions can correctly predict upcoming price decline, a positive return through short selling may not be guaranteed.

4.4 Impact of incentive slope and institutions' delegated investments

Section 4.3 studies the impact of benchmarking, or the penalty term, in the *QB* contract, on information acquisition and price informativeness, etc. This section turns to the impact of the incentive term.

Early studies, such as Stoughton (1993) and Admati and Pfleiderer (1997), study this issue with a representative institutional investor when the price is exogenously determined. They find that the incentive slope provides no incentive for information acquisition. More recent studies explore this subject under an endogenous price framework and reveal that the incentive slope has an incentive effect on information acquisition (Kyle et al. 2011; Huang et al. 2020). This section extends the literature to a perfectly competitive market with countless (continuous) institutions, whose incentive slopes may vary. That is, the exogenous and endogenous price models are incorporated into a unified framework, in which we investigate the mechanism for the effect of incentive slope.

Huang et al. (2020) divide investors in the market into two categories: institutional investors with information acquisition capability that make delegated investments, and retail investors who make direct investment for themselves by getting information from the market price only. Huang et al. (2020) argue that increased institutionalization—as indicated by the proportion of assets managed by institutions with information acquisition advantages—helps bring more information into the market, thereby improving price informativeness. Is the improved price informativeness due to institutions' informed trading caused by delegated investments or by institutions' information acquisition capability? Furthermore, in delegated investment, what role do different forms of contract play in impacting price and price informativeness? Huang et al. (2020) or others do not provide answers to these questions.

In order to explore these issues, this paper assumes that all investors in the market have the capability of acquiring information and thus conducting informed trading. These informed investors belong to one of two groups: those making direct investment for managing their own wealth, and institutional investors making delegated investments for managing others' wealth. Assume that the proportion of institutions undertaking delegated investment is τ, and the incentive slope of institutional investors is η. The informed investors who manage their own money can be regarded as non-benchmarked investors with an incentive slope of 1. Therefore, the modelling framework in Sect. 2 still applies, and the average incentive slope is $\overline{\eta} = 1/(\tau/\eta + 1 - \tau)$. The larger the proportion of institutions undertaking delegated investment (τ), the lower the $\overline{\eta}$.

4.4.1 Effect on information acquisition

Theorem 6 *When the average incentive slope increases, the information precision acquired by institutional investors rises, i.e.,* $\partial q_i/\partial\overline{\eta} > 0$*and* $\partial\overline{q}/\partial\overline{\eta} > 0$*. Keeping the average incentive slope constant, the change in an individual institution's incentive slope has no effect, i.e.,* $\partial q_i/\partial\eta_i = 0$.

Theorem 6 states that when $\overline{\eta}$ stays unchanged, the information acquisition of an institution is not affected by its own incentive slope. Only when the average incentive slope $\overline{\eta}$ increases will an institution's information acquisition effort increase. This is because the shared wealth for the institution is

$$\eta_i W_i = \eta_i + \eta_i\hat{\theta}_i(X - P) = \eta_i + \frac{\big(\hat{\mu}_{Xi} - P\big)(X - P)h_i}{\rho} \tag{21}$$

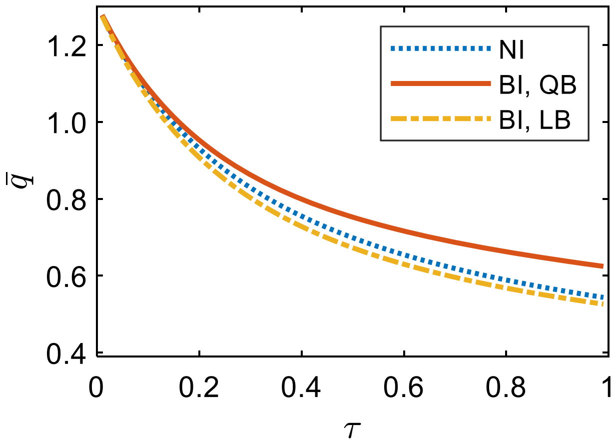

Fig. 7 Impact of τ on average information precision

When η_i increases, the institution can always reduce its risk exposure to maintain $\eta_i\hat{\theta}_i = \frac{(\hat{\mu}_{Xi}-P)}{\rho}h_i$ unchanged. The risk brought by increasing η_i is offset by the institution's investment strategy,[9] so it has no incentive to spend on improving information precision.

When $\overline{\eta}$ increases, institutions take greater risks while gaining a larger share of the wealth. While an institution can hedge its risk caused by the change in η_i, it cannot hedge the risk caused by the change in $\overline{\eta}$, because $\overline{\eta}$ impacts the asset price and price informativeness. In order to reduce the risk, an institution must improve accuracy in its information acquisition, and the average information precision thus improves. Therefore, only when the average incentive slope $\overline{\eta}$ increases can institutional investors' information acquisition efforts be improved.

The incentive slope of a single institution (η_i) does not affect $\overline{\eta}$. However, when the proportion of institutions undertaking delegated investment (τ) increases, $\overline{\eta}$ decreases. Based on Theorems 4 and 6, we can reach the following Corollary.

Corollary 2 *Increasing the proportion of institutions undertaking delegated investment will reduce information acquisition effort of individual institutions and lower the average information accuracy. Linear benchmarked contracts discourage institutions from acquiring information, resulting in a further decline in average information accuracy, while nonlinear benchmarked contracts can alleviate this problem.*

Figure 7 shows the average information accuracy as a function of the proportion of institutions undertaking delegated investment when the institutions are under different forms of contracts: non-benchmark investors (NI), benchmarked investors with the LB contract, or benchmarked investors with the QB contract. The parameters for Fig. 7 are the same as those for Fig. 1, where $\eta = 0.3$ and $\gamma = 0.3$. Corollary 2 and Fig. 7 illustrate that, compared to direct investment, delegated investment of

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[9] Huang et al. (2020) consider transaction frictions, which prevents institutions from reducing investment position proportionally to the growth of the incentive slope. This paper does not consider such frictions.

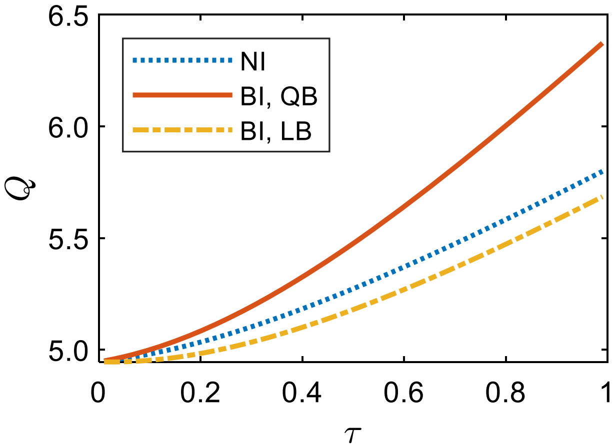

Fig. 8 Impact of τ on price informativeness

institutions will reduce their information acquisition efforts; the greater the proportion of institutions involved in delegated investment, the lower the average information accuracy. Intuitively, compared to direct investment by informed investors, delegated investment enables institutions to share risks with their clients, so institutions' zeal for information acquisition declines. Under the nonlinear contract QB, due to the client's requirement for the institution to obtain a positive rate of return and the threat of penalty, the declining trend of institutional information acquisition efforts is curbed. The QB contract thus can alleviate the problem of diminishing information acquisition efforts in delegated investments.

4.4.2 Effect on price informativeness

When $\overline{\eta}$ increases, average information precision improves, positively affecting price informativeness. In the meantime, the effective risk aversion $\rho\overline{\eta}$ increases, posing a negative effect on price informativeness. The impact of $\overline{\eta}$ on price informativeness thus depends on the relative strength of the two opposing effects.

Theorem 7 *The effect of* $\overline{\eta}$*on price informativeness satisfies* $\frac{\partial Q}{\partial \overline{\eta}} = \frac{2\overline{q}^2}{\rho^2\overline{\eta}^3\sigma_Z^2}\left(\frac{\partial \overline{q}}{\partial \overline{\eta}}\frac{\overline{\eta}}{\overline{q}} - 1\right)$. *Price informativeness increases with* $\overline{\eta}$*when* $\frac{\partial \overline{q}}{\overline{q}} > \frac{\partial \overline{\eta}}{\overline{\eta}}$*and decreaseshen* $\frac{\partial \overline{q}}{\overline{q}} < \frac{\partial \overline{\eta}}{\overline{\eta}}$.

When $\overline{\eta}$ rises, if the rate of change of $\overline{q}$ is greater (less) than the rate of change of $\overline{\eta}$, the positive (negative) effect of $\overline{\eta}$ on price informativeness dominates, and thus price informativeness improves (diminishes). Numerical demonstration in Fig. 8 shows that the negative effect of $\overline{\eta}$. on price informativeness is stronger than the opposing effect.[10]

Price informativeness Q changing inversely with $\overline{\eta}$ indicates that the rate of change of $\overline{q}$ is less than that of $\overline{\eta}$, which may be caused by two factors. Intuitively, the risk

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[10] Under the CARA expected utility, it can be proved that $\partial Q/\partial\overline{\eta} < 0$, *i.e.*, price informativeness Q changes inversely with $\overline{\eta}$. See Appendix B for details.

brought by a higher average incentive slope may be partially, although imperfectly, hedged by institutions' investment strategy. The mitigated risk retards their pursue of precise information. Moreover, information acquisition cost also contributes to the fact that the rate of change of $\overline{q}$ is lower than that of $\overline{\eta}$. Therefore, the price informativeness is a decreasing function of the average incentive slope $\overline{\eta}$.

Based on Corollary 1 and Theorem 7, we can reach the following result: as the proportion of institutions undertaking delegated investment increases, price informativeness improves. LB contracts retard the upward trend of price informativeness, while QB contracts can accelerate the increase in price informativeness.

The change of price informativeness with τ is shown in Fig. 8, whose parameters are the same as those for Fig. 7. Figure 8 demonstrates that delegated investment by institutions helps improve price informativeness. Intuitively, delegated investment hinders the information acquisition efforts of institutions, but compared to direct investment, delegated investment enables institutions to share risks with clients. As the investment risk they bear is reduced, their effective degree of risk aversion decreases, which prompts institutions to make more active investments. The ability of the market price to transmit information is thus enhanced, so price informativeness increases.

Therefore, improved price informativeness brought by institutionalization is caused largely by delegated investments. If informed investors make direct investment only, the improvement in price informativeness would be greatly attenuated.

In addition, the form of delegation contracts matters. Price informativeness increases the most under the nonlinear contract. Under the QB contract, due to its positive return benchmark, institutions have to work harder in information acquisition, so the price informativeness will increase faster. Therefore, the nonlinear benchmarked contract can strengthen the contribution of delegated investment to improving price informativeness, as shown in Fig. 8.

4.4.3 Effect on investment strategy and equilibrium price

Existing research regarding the impact of compensation contracts on prices is usually based on a representative institutional investor. If distinctions among institutional investors are considered—such as different degrees of risk aversion or different delegation contracts—then how would delegated investment and contracts affect the price of risky assets? Huang et al. (2020) consider information acquisition and investigate how institutionalization affects price informativeness and volatility, but do not examine how institutions' contracts may influence the price. Extant studies provide no analytical results regarding the impact of the incentive slope on price fluctuation, nor do they unpack the mechanism of such an impact in terms of delegated investment strategies or price determination. This section conducts some exploration into this subject. Within our modelling framework, we have the following result:

Theorem 8

1. *The optimal investment strategy satisfies*

$$E\left(\hat{\theta}_i\right) = \frac{\overline{\eta}\mu_Z}{\eta_i \overline{h}} h_i. \tag{22}$$

When the average incentive slope $\overline{\eta}$*and penalty intensity* γ_i*stay constant, the optimal investment* $E\left(\hat{\theta}_i\right)$*is decreasing in* η_i*, i.e.,* $\partial E\left(\hat{\theta}_i\right)/\partial \eta_i < 0$.

2. *The equilibrium risky asset price satisfies*

$$E(P) = \mu_X - \frac{1}{H}\rho\overline{\eta}\mu_Z \tag{23}$$

$$\frac{\partial E(P)}{\partial \overline{\eta}} = \frac{\rho\mu_Z}{\overline{h}}\left(\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} - 1\right). \tag{24}$$

When $\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} > 1$, $\frac{\partial E(P)}{\partial \overline{\eta}} > 0$; when $\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} < 1$, $\frac{\partial E(P)}{\partial \overline{\eta}} < 0$.

Keeping $\overline{\eta}$ constant, changing the individual incentive slope η_i has no effect on information acquisition, but affects an institution's investment decision. In order to counter the higher risk caused by η_i, the institution takes a more conservative investment strategy by reducing risk exposure. $E\left(\hat{\theta}_i\right)$ is hence decreasing in η_i. This result is consistent with Admati and Pfleiderer (1997) and Kyle et al. (2011). When higher incentive slopes are prevalent, $\overline{\eta}$ increases, and $E(P)$ decreases.

$\overline{\eta}$ affects aggregate demand for the risky asset and thus its equilibrium price in two ways. On one hand, when $\overline{\eta}$ increases, as per Theorem 6, institutions generally raise information precision, thus $\overline{q}$ is higher. With improved information, the uncertainty regarding the risky asset's payoff decreases. To obtain greater utility, the equilibrium price is therefore under upward pressure. On the other hand, as $\overline{\eta}$ rises, the effective risk aversion of institutions increases. Then institutions tend to cut investment in the risky asset, which exerts downward pressure on its equilibrium price.

The overall impact of $\overline{\eta}$ on the equilibrium price depends on the relative strength of the two opposing effects. When the aggregate informational efficiency increases faster than the average incentive slope, i.e., $\partial H/H > \partial\overline{\eta}/\overline{\eta}$, institutions are keen to chase returns; a higher demand elevates the equilibrium price. In contrast, when $\partial H/H < \partial\overline{\eta}/\overline{\eta}$ institutions tend cut back their risk exposure as $\overline{\eta}$ rises to hedge increased risk. A reduced aggregate demand lowers the price. Numerical demonstrations as depicted in Fig. 9 show that the negative effect dominates so that $E(P)$ is decreasing in $\overline{\eta}$.[11]

When an institution has no market power and cannot influence the price, it is regarded as a price taker. In contrast, the price is endogenous when an institution has market power or when the collective change in institutions' investment decisions is sufficient to move the market price. From the perspective of compensation contracts, the issue is whether changing an individual contract's parameters can affect market average contract parameters. If so, the price is endogenous to the individual institution. In this paper, it is evident that changing the proportion of institutions undertaking delegated investment or changing the form of institutions' compensation contracts can affect the risky asset price.

Figure 9 illustrates the relationship between the risky asset price and the proportion of institutions undertaking delegated investment under different contracts:

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[11] Under the CARA expected utility, it can be proved that $\partial E(P)/\partial\overline{\eta} < 0$ always holds; see Appendix B for details.

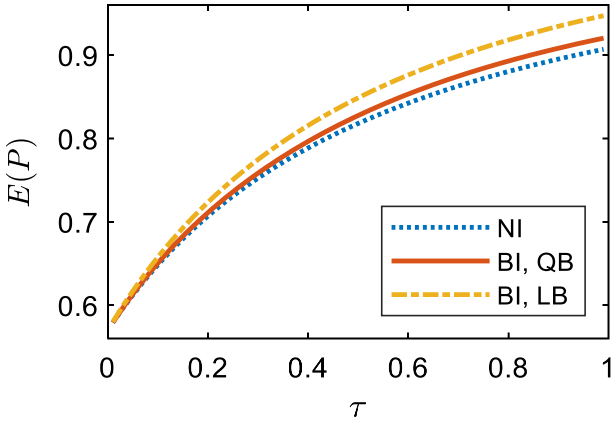

Fig. 9 Impact of τ on expected price

non-benchmarked (NI), linear benchmarked (LB), or nonlinear benchmarked (QB). The parameter setting in Fig. 9 is the same as that in Fig. 7. As the proportion of institutions participating in delegated investment rises, the risks borne by institutional investors as informed investors decrease. Their degree of effective risk aversion declines, so their investment strategies become more aggressive, thereby pushing up the price of the risky asset. The risky asset price thus increases with the proportion of institutions undertaking delegated investment. The form of delegation contract can further amplify or weaken the impact of delegated investments on the risky asset price. As shown in Fig. 9, the price increases less under the QB contract than under the LB contract.

In summary, delegated investment increases the zeal of institutional investment, raises price informativeness, and elevates the risky asset price. Different benchmark choices affect institutions' information acquisition, and thus impact price informativeness and the asset price. The QB contract alters institutions' behaviors through its penalty term, motivating them to work hard to obtain information and make active investment. The QB contract thus strengthens the contribution of delegated investment to improving price informativeness and restraining price increase.

5 Optimal performance-based QB contract

The previous section has demonstrated that institutional delegated investment and particularly the QB contract have advantages in enhancing price informativeness and containing price inflation. Then an intriguing question is: what is the form of the optimal QB contract, if it exists? This section explores this issue. Given the performance-based contract (14), the investor wants to find the optimal contract parameters to maximize their expected utility. In order to simplify the analysis, we first assume that the investor is risk-neutral following Kyle et al. (2011) and examine the optimal *QB* contract. Then we investigate the case when the investor is risk averse.

5.1 When investor is risk neutral

When an investor is risk neutral, their expected utility is

$$U_P(\eta_i, \gamma_i) = E[W_i - \omega_i] = E[(1-\eta_i)(1+\theta_i(X-P)) + \gamma_i\Big(X-P)^2 - a(\eta_i, \gamma_i)\Big] \tag{25}$$

where $W_i = 1+\theta_i(X-P), h = 1$, and ω_i is as in Eq. (14). Unlike in Eq. (14), the fixed compensation $a(\eta_i, \gamma_i)$ is endogenously determined and ensures that the institution's participation constraint is satisfied. The parameters in the optimal contract satisfy $\left(\hat{\eta}_i, \hat{\gamma}_i\right) = \arg \underset{(\eta_i, \gamma_i)}{max}\, U_P(\eta_i, \gamma_i)$. Assume that the institutional investor has a reserved utility u_{0i}. Then $\left(\hat{\eta}_i, \hat{\gamma}_i\right)$ satisfies the following constraints:

1. $\hat{\gamma}_i \geq 0$ and $0 \leq \hat{\eta}_i \leq 1$;
2. when $\eta_i \neq 0$, the optimal portfolio satisfies $\hat{\theta}_i = \frac{(\hat{\mu}_{Xi}-P)}{\rho\eta_i} h_i$;
3. when $\eta_i \neq 0$, the optimal information precision $\hat{q}_i$ satisfies Eq. (18);
4. the institution's participation constraint is satisfied, i.e., $U_i \geq u_{0i}$.

According to the participation constraint,

$$a(\eta_i, \gamma_i) \geq u_{0i} - \eta_i + \frac{1}{2\rho}\ln\left(\frac{h_i}{h_i - 2\rho\gamma_i}\right) + k\left(\hat{q}_i\right) - \frac{1}{2\rho}Ah_i + \frac{1}{2\rho}, \tag{26}$$

where $A = \frac{1}{H^2}\left(\rho^2\overline{\eta}^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + \overline{q}\right)$. Therefore $\left(\hat{\eta}_i, \hat{\gamma}_i\right)$ satisfies

$$\begin{aligned} U_P(\eta_i, \gamma_i) = 1 &+ \frac{1-\eta_i}{\rho\eta_i} E\left(\left(\hat{\mu}_{Xi} - P\right)h_i(X-P)\right) + \gamma_i E\left((X-P)^2\right) \\ &- u_{0i} - \left[\frac{1}{2\rho}\ln\left(\frac{h_i}{h_i - 2\rho\gamma_i}\right) + k\left(\hat{q}_i\right) - \frac{1}{2\rho}Ah_i + \frac{1}{2\rho}\right], \end{aligned} \tag{27}$$

where

$$\begin{aligned} E\left(\left(\hat{\mu}_{Xi} - P\right)(X-P)\right) = \frac{(h_i - H)}{h_i} E\left((X-P)^2\right) - \left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)\frac{\rho\overline{\eta}\mu_Z}{Hh_i} \\ + \frac{\rho\overline{\eta}}{Hh_i}\left(\frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}} + \rho\overline{\eta}\right)\left(\mu_Z^2 + \sigma_Z^2\right), \end{aligned} \tag{28}$$

$$E\left[(X-P)^2\right]=\frac{-2\rho\overline{\eta}\mu_Z}{H^2}\left(\frac{\mu_X}{\sigma_X^2}+\frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)-\left(\frac{1}{H}\left(\frac{\mu_X}{\sigma_X^2}+\frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)\right)^2$$
$$+\frac{\mu_X^2+\sigma_X^2}{H^2\sigma_X^4}+\left(\frac{1}{H}\left(\frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}}+\rho\overline{\eta}\right)\right)^2\left(\mu_Z^2+\sigma_Z^2\right). \tag{29}$$

As assumed in Sect. 2, there exists a continuum of institutional investors in the market. Changing contract parameters for a single institution does not affect the average information accuracy ($\overline{q}$), average penalty intensity ($\overline{\gamma}$), or average incentive slope ($\overline{\eta}$), and $\frac{\partial h_i}{\partial \eta_i}=\frac{\partial \hat{q}_i}{\partial \eta_i}=0$. Taking partial derivative with respect to η_i to the investor's utility, we get

$$\frac{\partial U(\eta_i,\gamma_i)}{\partial \eta_i}=-\frac{1}{\rho\eta_i^2}h_iE\left(\left(\hat{\mu}_{Xi}-P\right)(X-P)\right). \tag{30}$$

Assume $k(q)=\frac{1}{2}q^2$. Plugging $\frac{\partial h_i}{\partial \gamma_i}=\frac{(h_i-2\rho\gamma_i)^{-2}}{1+\frac{1}{2\rho}[(h_i-2\rho\gamma_i)^{-2}-h_i^{-2}]}$ into the first order condition of the investor's utility, we have the following results.

Theorem 9

1. *When* $E\left(\left(\hat{\mu}_{Xi}-P\right)(X-P)\right)\leq 0$, *the optimal information precision satisfies* Eq. (18), $\hat{\eta}_i=1$, *the optimal contract satisfies* $\omega_i=a_i+W_i-\hat{\gamma}_i(R_B)^2$, *and* $\hat{\gamma}_i$ *satisfies*

$$E\left[(X-P)^2\right]=\frac{1}{h_i-2\rho\gamma_i} \tag{31}$$

2. *When* $E\left(\left(\hat{\mu}_{Xi}-P\right)(X-P)\right)>0$, *if the investor delegates investment to the institution, the optimal information precision satisfies* Eq. (18). *The theoretically optimal incentive slope does not exist; the investor's expected utility is maximized when* η_i *approaches zero.*
3. *When* $E\left(\left(\hat{\mu}_{Xi}-P\right)(X-P)\right)>0$, *if the investor makes investment decision based on the information collected by the institution (i.e.,* $\eta_i=0$*), the optimal contract is* $\omega_i=a_i-\gamma_iW_{0i}(R_B)^2$ *and the optimal information precision satisfies*

$$k'(q_i)=\frac{1}{2\rho}\left(\frac{1}{h_i-2\rho\gamma_i}-\frac{1}{h_i}\right)-\frac{\gamma_i}{h_i-2\rho\gamma_i}B+\frac{\gamma_i}{(h_i-2\rho\gamma_i)^2}[Bh_i-1], \tag{32}$$

where $B = \frac{1}{H^2}\left(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\right) + \left(\frac{\rho\overline{\eta}\mu_Z}{H}\right)^2$. The optimal penalty intensity satisfies

$$E_1\left[\left(\hat{\mu}_{Xi} - P\right)^2\right] = \left(\frac{\gamma_i}{h_i}\left(\frac{1}{h_i} - \frac{1}{h_i - 2\rho\gamma_i}\right) - \frac{A}{2\rho} + k'\left(q_i\right)\right)\frac{\partial q_i}{\partial \gamma_i} + \frac{1}{h_i - 2\rho\gamma_i} - \frac{1}{h_i} - \gamma_i\frac{\partial E_1\left[\left(\hat{\mu}_{Xi} - P\right)^2\right]}{\partial \gamma_i}, \tag{33}$$

where $E_1\left[\left(\hat{\mu}_{Xi} - P\right)^2\right] = \left(1 - \frac{H}{h_i}\right)^2 E\left[(X - P)^2\right] + \frac{q_i}{(h_i)^2} + \left(\frac{\rho\overline{\eta}}{h_i}\right)^2\left(\sigma_Z^2 + \mu_Z^2\right)$.

For a risk-neutral investor, the optimal incentive slope depends on whether the institution can correctly predict the sign of the risky asset's return. Since the investor and the institution engage in a game relationship, the investor wants to select the contract parameters in advance based on their perception of the institution's capability to maximize their expected utility. If the institution regularly misjudges the market by calling the wrong direction (i.e., $E\left(\left(\hat{\mu}_{Xi} - P\right)(X - P)\right) \leq 0$), it will make wrong investment decisions that would lead to sizeable losses. At the end, both parties will share the investment return as per the contract: the institution gets $\eta_i W_i$, and the investor receives $(1 - \eta_i)W_i$, including losses caused by the institution's misjudgment. The smaller the η_i, the more aggressive the institution's investment strategy. Given this consideration, the investor wants to protect themselves by selecting the optimal incentive slope as $\hat{\eta}_i = 1$: the institution bears the full consequences of its regular misjudgments, and the investor will gain maximum utility at $W_i - \omega_i = \gamma_i(X - P)^2 - a_i$.

If, on average, the institution has accurate predictions on the sign of the risky asset's return (i.e., $E\left(\left(\hat{\mu}_{Xi} - P\right)(X - P)\right) > 0$), the risk-neutral investor, who cares about investment return only, expects the institution to take an unlimited position. However, since the institution is risk averse, the investor will reduce η_i to lower the institution's effective risk aversion $\rho\eta_i$ in order to motivate it to take an aggressive position. Because there are continuous institutions in the market, the investment decision of a single institution does not affect the risky asset's price, so the arbitrage opportunity always exists. In order to obtain an even higher return, the investor would keep reducing the incentive slope to encourage the institution's risk-taking. The smaller η_i is, the bigger the institution's risk exposure $\hat{\theta}_i = \frac{(\hat{\mu}_{Xi} - P)}{\rho\eta_i}h_i$, and the greater the investor's expected utility.

When the institution is able to correctly predict the sign of the risk asset's return on average, the investor can also choose to make investment decisions themselves based on the information provided by the institution. In this situation, the incentive slope $\eta = 0$. However, the institution's information acquisition behavior would differ from the situation when $\eta \neq 0$, since the institution cannot obtain its reward from sharing the end-of-period wealth.

It is worthwhile to note that the above discussions in the case of a risk-neutral investor differ from Kyle et al. (2011) in which the investment strategy of a single institution can impact the asset price. Because of this difference, in their study the extreme cases of optimal incentive slope as 1 or (close to) 0 do not arise since the arbitrage opportunity will disappear when even a single institution takes an aggressive position.[12]

In summary, when the client is a risk-neutral investor, the optimal contract takes one of the two forms. First, the contract takes the form as in Eq. (14). In this situation, the incentive slope is 1. Second, the optimal contract is in the form of a constant minus a quadratic term, similar to the nonlinear contract in Stoughton (1993). The difference is that Stoughton (1993) assumes that investors are risk-averse and the price is exogenously given, while this section assumes investors are risk-neutral, as in Kyle et al. (2011). In another scenario, the theoretically optimal incentive slope does not exist, but the investor's expected utility is maximized when η_i approaches zero.

5.2 When investor is risk averse

Next, we turn to the case when the investor is risk averse. We explore the optimal incentive slope and optimal penalty intensity in the *QB* contract that maximize the investor's expected utility.

When the investor is risk averse, assume the degree of risk aversion is ρ_1 and the utility function is CARA. The investor's expected utility function satisfies:

$$\begin{aligned} U_P(\eta_i, \gamma_i) &= -E\big[\exp(-\rho_1(W_i - \omega_i))\big] \\ &= -E\big[\exp - \rho_1(1-\eta_i)(1+\theta_i(X-P)) - \rho_1\gamma_i\big((X-P)^2 + \rho_1 a(\eta_i, \gamma_i)\big)\big] \\ &= -\exp(-\rho_1(1-\eta_i) + \rho_1 a(\eta_i, \gamma_i)) \\ &\quad \cdot E\left[\exp\left(\frac{-\rho_1(1-\eta_i)}{\rho\eta_i}\big(\hat{\mu}_{Xi} - P\big)h_i(X-P) - \rho_1\gamma_i\big(X-P'\big)^2\right)\right] \end{aligned} \tag{34}$$

The risk-averse investor's expected utility satisfies the same constraints as for the risk-neutral investor, where $a(\eta_i, \gamma_i)$ satisfies Eq. (26) , the optimal portfolio satisfies $\hat{\theta}_i = \frac{(\hat{\mu}_{Xi}-P)}{\rho\eta_i}h_i$, and the optimal information precision $\hat{q}_i$ satisfies Eq. (18).

Note

$$\begin{aligned} \big(\hat{\mu}_{Xi} - P\big)h_i = {} & \left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)\left(1 - \frac{h_i}{H}\right) + q_i\varepsilon_i - \frac{1}{\sigma_X^2}\left(1 - \frac{h_i}{H}\right)X \\ & + \frac{\rho\overline{\eta}}{\overline{q}}\left(h_i - \frac{\overline{q}^2}{\sigma_Z^2\rho^2\overline{\eta}^2} - \frac{h_i}{H\sigma_X^2}\right)Z, \end{aligned} \tag{35}$$

$$X - P = -\frac{1}{H}\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right) + \frac{1}{H\sigma_X^2}X + \frac{1}{H}\left(\frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}} + \rho\overline{\eta}\right)Z. \tag{36}$$

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[12] There is an implicit assumption in Sects. 2, 3, and 4 that the incentive slope in the contract of individual institutions is non-zero. The information acquisition when the incentive slope is 0, as examined in Sect. 5.1, is thus an extension to the results in Sects. 2, 3, and 4.

When S follows the normal distribution $N(0, \Sigma_S)$, according to Brunnermeier (2001, page 64),

$$E\Big[\exp\Big(S^T AS + K^T S + L\Big)\Big] = |I - 2\Sigma A|^{-\frac{1}{2}} \exp\Big(\frac{1}{2} K^T (I - 2\Sigma A)^{-1} \Sigma K + L\Big).$$

Let random vector $S = (X - \mu_X, Z - \mu_Z, \varepsilon)$, then $S \sim N(0, \Sigma_S)$, where $\Sigma_S = disg\Big\{\sigma_X^2, \sigma_Z^2, \frac{1}{q_i}\Big\}$. Substitute Eqs. (35) and (36) into Eq. (34) and take expectation, then the investor's expected utility is

$$\begin{aligned} U_P(\eta_i, \gamma_i) = {} & -|I - 2\Sigma_S D|^{-\frac{1}{2}} \exp(-\rho_1(1 - \eta_i) + \rho_1 a(\eta_i, \gamma_i) + d_0) \\ & \times \exp\Big[\frac{1}{2}(d_1\ d_2\ d_3)(I - 2\Sigma_S D)^{-1} \Sigma_S (d_1\ d_2\ d_3)^T\Big] \end{aligned} \tag{37}$$

where I is the identity matrix of order 3 and $D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$,

$$d_{11} = \frac{\rho_1(1 - \eta_i)}{\rho\eta_i} \frac{1}{\sigma_X^2} \left(1 - \frac{h_i}{H}\right) \frac{1}{H\sigma_X^2} - \rho_1\gamma_i \left(\frac{1}{H\sigma_X^2}\right)^2,$$

$$\begin{aligned} d_{22} = {} & -\frac{\rho_1(1 - \eta_i)}{\rho\eta_i} \frac{1}{H} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right) \frac{\rho\overline{\eta}}{\overline{q}} \left(h_i - \frac{\overline{q}^2}{\sigma_Z^2 \rho^2 \overline{\eta}^2} - \frac{h_i}{H\sigma_X^2}\right) \\ & - \rho_1\gamma_i \left(\frac{1}{H} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right)\right)^2, \end{aligned}$$

$$\begin{aligned} d_{12} &= d_{21} \\ &= \frac{\rho_1(1 - \eta_i)}{2\rho\eta_i} \frac{1}{H} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right) \frac{1}{\sigma_X^2} \left(1 - \frac{h_i}{H}\right) - \rho_1\gamma_i \frac{1}{H\sigma_X^2} \frac{1}{H} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right) \\ &\quad - \frac{\rho_1(1 - \eta_i)}{2\rho\eta_i} \frac{\rho\overline{\eta}}{\overline{q}} \left(h_i - \frac{\overline{q}^2}{\sigma_Z^2 \rho^2 \overline{\eta}^2} - \frac{h_i}{H\sigma_X^2}\right) \frac{1}{H\sigma_X^2}, \end{aligned}$$

$$d_{13} = d_{31} = -\frac{\rho_1(1 - \eta_i)}{2\rho\eta_i} \frac{q_i}{H\sigma_X^2}, d_{23} = d_{32} = -\frac{\rho_1(1 - \eta_i)}{2\rho\eta_i} \frac{q_i}{H} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right), d_{33} = 0,$$

$$d_1 = \frac{-\rho_1(1 - \eta_i)}{\rho\eta_i} \left[\frac{2h_i \rho\overline{\eta}\mu_Z}{H^2 \sigma_X^2} - \frac{\rho\overline{\eta}\mu_X}{H\sigma_X^2}\right] - \rho_1\gamma_i \frac{2\rho\overline{\eta}\mu_Z}{H^2 \sigma_X^2},$$

$$d_2 = \frac{-\rho_1(1 - \eta_i)}{\rho\eta_i} \left[\frac{h_i \rho\overline{\eta}\mu_Z}{H^2} \left(\frac{\overline{q}}{\sigma_Z^2 \rho\overline{\eta}} + \rho\overline{\eta}\right) + \frac{(\rho\overline{\eta})^2 \mu_Z}{H\overline{q}} \left(h_i - \frac{\overline{q}^2}{\sigma_Z^2 \rho^2 \overline{\eta}^2} - \frac{h_i}{H\sigma_X^2}\right)\right]$$

$$- \rho_1 \gamma_i \frac{2\rho\overline{\eta}\mu_Z}{H^2}\left(\frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}} + \rho\overline{\eta}\right),$$

$$d_3 = \frac{-\rho_1(1-\eta_i)}{\rho\eta_i}\frac{q_i\rho\overline{\eta}\mu_Z}{H}, \quad and\; d_0 = \frac{-\rho_1(1-\eta_i)}{\rho\eta_i}\left(\frac{\rho\overline{\eta}\mu_Z}{H}\right)^2 h_i - \rho_1\gamma_i\left(\frac{\rho\overline{\eta}\mu_Z}{H}\right)^2.$$

Make Eq. (26) an equation, substitute Eqs. (26) and (18) into the investor's expected utility function Eq. (37). Search for (γ_i, η_i) in the ranges $\hat{\gamma}_i \geq 0$ and $0 \leq \hat{\eta}_i \leq 1$ that maximizes the expected utility. Then plug the optimal $\hat{\gamma}_i$ and $\hat{\eta}_i$ back into (Eq. 26) to obtain $a(\eta_i, \gamma_i)$ of the optimal contract. The investor's optimal QB contract is thus obtained.

When the investor is risk-averse, their expected utility function Eq. (37) takes an extremely complex form so that the optimal contract parameters can only be solved numerically, but the main results in this paper are still applicable. This is because when the contract is given, it is exogenous to the institution. The institution will acquire information and invest under the given contract. As long as the optimal contract parameter $\left(\hat{\gamma}_i, \hat{\eta}_i\right)$ of an institution is non-zero, the results in Sect. 4 regarding the impact of QB contract on institutional information acquisition, risky asset price, and price informativeness still apply under the optimal contract.

This study on the nonlinear benchmark provides some fresh ideas that meet practical needs and thus have good application prospects. First, Ma et al. (2019) document that 78% of active funds tie manager compensation to a benchmark. Sockin and Xiaolan (2023) argue that contract commonality—the phenomenon that active managers have a similar compensation structure across funds with similar loadings on each—amplifies the distortions from active managers' benchmark hedging demand and lowers the price of risk. As demonstrated in this study, contracts with a nonlinear benchmark can mitigate the negative impacts of contract commonality by providing meaningful and constructive contract alternatives.

Second, investors' diverse needs can be met with diverse contract options. A delegation contract with a non-negative benchmark is able to suit the need of investors who are extremely risk-averse and thus demand positive investment returns. Contracts with a quadratic benchmark can motivate institutions to acquire more accurate information and strive for positive returns. In this process, the enhanced informativeness of asset prices is in the interest of market participants and the public.

Finally, the quadratic benchmark proposed in this study is simple to implement. Although short selling restrictions would limit the effectiveness of the quadratic benchmark in bear markets, they are not common in developed stock markets. Overall, the quadratic benchmark has features appealing to all parties: institutions are subject to lower penalties, investors may enjoy better fund performance due to institutions' elevated efforts in information acquisition, and market regulators would value improved market information efficiency.

6 Conclusion

In order to address suboptimal information acquisition and deteriorated price informativeness caused by linear benchmarking in delegation contracts, this study proposes a novel benchmark that is a quadratic function of benchmark portfolio return. Comparisons demonstrate the compelling advantages of the nonlinear benchmarked contract in information acquisition incentive and price informativeness over the conventional linear benchmarked contract. The nonlinear benchmarked contract can mitigate the inflated price and volatility caused by institutions' pegging to the benchmark under linear benchmarking. The proposed contract also improves institutions' expected utility and reduces investors' fixed delegation costs when the penalty intensity is relatively low. Moreover, it can appease investors' resentment against rewarding negative investment returns.

Our model assumes a continuum of institutional investors whose compensation contracts can be heterogenous. This framework allows us to simultaneously examine institutional information acquisition and investment strategy in situations with an endogenous or exogenous price. The results show that only increasing the average incentive slope can motivate institutions to acquire information, while increasing the incentive slope for an individual institution alone has no incentive effect. Further analysis finds that the participation of informed traders in delegated investment helps to improve price informativeness. The larger the proportion of institutions undertaking delegated investment, the higher the price informativeness. The delegation contract with a quadratic benchmark can further strengthen the contribution of delegated investment to improving price informativeness, and mitigate the price inflation caused by institutions' excessive holding of the benchmark portfolio under the linear benchmarked contract.

Section 5 conducts an exploration of the optimal performance-based contract with a quadratic benchmark that maximizes the investor's expected utility, and discusses its application prospects. Considering the limitations of existing homogeneous delegation contracts, the nonlinear contract in this study offers a meaningful alternative. The proposed contract can meet investors' desire for positive investment returns, help improve institutions' efforts to obtain information, and enhance price informativeness. The quadratic benchmark is easy to implement and practical. It can play an important role in serving investors' diverse needs and motivating institutions to leverage their information advantages.

In future research, we will investigate compensation contract design when transaction friction exists, and study the impact of such compensation contracts on information acquisition and asset prices.

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Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

Appendix A: Proof of Theorems 1–9

Proof of Theorem1 The institution makes its investment decision upon receiving its private signal regarding the risky asset's payoff and observing the price. Denote the institution's conditional expected utility by $\widehat{U}_i = E_2(-\exp(-\rho C_i)|Y_i, P)$. Let $y = X|Y_i, P$, then $y \sim N\left(\hat{\mu}_{Xi}, \frac{1}{h_i}\right)$. In order to ensure the convergence and concavity of $\widehat{U}_i$, let $h_i - 2\rho\gamma_i > 0$, then

$$\widehat{U}_i = E_2(-\exp(-\rho C_i)|Y_i, P)$$

$$= E_2\Big(-\exp(-\rho\Big(a_i + \eta_i + \eta_i\theta_i(X - P) - \gamma_i\Big(X - P)^2 - k(q_i)\Big)\Big)|Y_i, P\Big)$$

$$= -\frac{\sqrt{h_i}}{\sqrt{2\pi}}\exp[-\rho a_i - \eta_i\rho + \rho k(q_i) + \rho\eta_i\theta_i P + \rho\gamma_i P^2]$$

$$\cdot \int_{-\infty}^{+\infty} \exp\Big[\rho\gamma_i y^2 - (\rho\eta_i\theta_i + 2\rho\gamma_i P)y\Big] \cdot \exp\Big(-\frac{(y - \hat{\mu}_{Xi})^2}{2}h_i\Big)dy$$

$$= -\frac{\sqrt{h_i}}{\sqrt{h_i - 2\rho\gamma_i}}\exp$$

$$\left[\frac{(2\rho\gamma_i P + \rho\eta_i\theta_i - \hat{\mu}_{Xi}h_i)^2}{2(h_i - 2\rho\gamma_i)} - \rho\eta_i - \rho a_i + \rho k(q_i) + \rho\eta_i\theta_i P + \rho\gamma_i P^2 - \frac{h_i}{2}\left(\hat{\mu}_{Xi}\right)^2\right]$$

$$\cdot \underbrace{\frac{\sqrt{h_i - 2\rho\gamma_i}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left[-\frac{h_i - 2\rho\gamma_i}{2}\left(y + \frac{2\rho\gamma_i P + \rho\eta_i\theta_i - \hat{\mu}_{Xi}h_i}{h_i - 2\rho\gamma_i}\right)^2\right]dy}_{=1}$$

$$= -\frac{\sqrt{h_i}}{\sqrt{h_i - 2\rho\gamma_i}}\exp$$

$$\left[\frac{(2\rho\gamma_i P + \rho\eta_i\theta_i - \hat{\mu}_{Xi}h_i)^2}{2(h_i - 2\rho\gamma_i)} - \rho a_i - \rho\eta_i + \rho k(q_i) + \rho\eta_i\theta_i P + \rho\gamma_i P^2 - \frac{h_i}{2}(\hat{\mu}_{Xi})^2\right]$$

Differentiate $\widehat{U}_i$ with respect to θ_i and let it equal to zero, and then simplify the result:

$$\frac{\left(2\rho\gamma_i P + \rho\eta_i\theta_i - \hat{\mu}_{Xi}h_i\right)\eta_i\rho}{(h_i - 2\rho\gamma_i)} + \rho\eta_i P = 0. \tag{A1}$$

Rearrangement of (A1) leads to Eq. (15). □

Proof of Theorem 2 Assume that the equilibrium price of the risky asset is a linear function of its payoff and its total supply as (Breugem and Buss 2019; Garcia and Vanden 2009).

$$P = a + bX - dZ, \tag{A2}$$

Rearranging (A2) gives $\frac{P-a+d\mu_Z}{b} = X - \frac{d(Z-\mu_Z)}{b}$, where $-\frac{d(Z-\mu_Z)}{b} \sim N\left(0, \frac{d^2\sigma_Z^2}{b^2}\right)$.

Since $Y_i = X + \varepsilon_i$, $\varepsilon_i \sim N\left(0, \frac{1}{q_i}\right)$, the conditional precision of payoff X by the risky asset is

$$h_i = (Var(X|Y_i, P))^{-1} = (Var(X))^{-1} + q_i + \left(Var\left(-\frac{d(Z-\mu_Z)}{b}\right)\right)^{-1} = \frac{1}{\sigma_X^2} + \frac{b^2}{d^2\sigma_Z^2} + q_i. \tag{A3}$$

By Bayes' rule, the conditional mean of X is

$$\begin{aligned}\hat{\mu}_{Xi} &= E(X|Y_i, P)\\ &= Var(X|Y_i, P)\left((Var(X))^{-1}\mu_X + q_i Y_i + \frac{P-a+d\mu_Z}{b}\left(Var\left(-\frac{d(Z-\mu_Z)}{b}\right)\right)^{-1}\right),\end{aligned}$$

and its simplification gives

$$\hat{\mu}_{Xi} = \frac{1}{h_i}\left(\frac{\mu_X}{\sigma_X^2} + q_i Y_i + \frac{b^2}{d^2\sigma_Z^2}\frac{P-a+d\mu_Z}{b}\right). \tag{A4}$$

The equilibrium price is determined by market clearing, that is, aggregate demand equals aggregate supply, or $\int_0^1 \hat{\theta}_i di = Z$. Plug the expressions of $\hat{\mu}_{Xi}$, h_i, and P into $\hat{\theta}_i$. in the market clearing condition:

$$\int_0^1 \frac{1}{\rho\eta_i}\left(\frac{\mu_X}{\sigma_X^2} + q_i Y_i + \frac{b(-a+d\mu_Z)}{d^2\sigma_Z^2} - P\left(\frac{1}{\sigma_X^2} + q_i + \frac{b(b-1)}{d^2\sigma_Z^2}\right)\right)di = Z. \tag{A5}$$

To simplify analysis, assume the same incentive slope for all institutions, i.e., $\eta_i = \overline{\eta}$ (different incentive slopes are allowed for a limited number of institutions, because they would not affect the value of $\int_0^1 \frac{1}{\eta_i}di$). Now we have $\int_0^1 \frac{1}{\eta_i}di = \frac{1}{\overline{\eta}}$, where $\overline{\eta}$ is the average incentive slope.

Note that $Y_i = X + \varepsilon_i$, $\int_0^{\nu} \varepsilon_i di = 0$, $\int_{\nu}^1 \varepsilon_i di = 0$. Let $\overline{q} = \int_0^{\lambda_Q} q_i^{BI} di + \int_{\lambda_Q}^1 q_i^{NI} di$. Let q_i^{BI} represent the information precision of an institutions subject to a benchmarked contract, and q_i^{NI} represent the information precision of an institution with a non-benchmarked contract. Plugging the conditions above into the market clearing condition (A5) gives

$$\frac{1}{\rho\overline{\eta}}\left(\frac{\mu_X}{\sigma_X^2} + \overline{q}X + \frac{b(-a+d\mu_Z)}{d^2\sigma_Z^2} - P\left(\frac{1}{\sigma_X^2} + \overline{q} + \frac{b(b-1)}{d^2\sigma_Z^2}\right)\right) = Z. \tag{A6}$$

Gather P to the left-hand side of the equation, and compare it to $P = a + bX - dZ$. The equality of the corresponding coefficients yields a set of three equations. Solving a, b, and d leads to Eq. (16) , where $H = \frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2} + \overline{q}$. ☐

Proof of Theorem 3 Plug the optimal portfolio $\hat{\theta}_i$ into $E_2(-\exp(-\rho C_i)|Y_i, P)$:

$$\begin{aligned} &\underset{\theta_i}{max}\ E_2(-\exp(-\rho C_i)|Y_i, P) \\ &\quad = -\left(1 - 2\rho\gamma_i\frac{1}{h_i}\right)^{-\frac{1}{2}}(\exp(-\rho a_i - \rho\eta_i + \rho k(q_i)) \cdot \exp\left(-\frac{1}{2}\left(\hat{\mu}_{Xi} - P\right)^2 h_i\right). \end{aligned} \tag{A7}$$

Let $g_i = h_i\left(\hat{\mu}_{Xi} - P\right)$. Plugging $Y_i = X + \varepsilon_i$ and $\frac{P - a + d\mu_Z}{b} = X - \frac{\rho\overline{\eta}}{\overline{q}}(Z - \mu_Z)$ into Eq. (A4) gives

$$\hat{\mu}_{Xi} = \frac{1}{h_i}\left(\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right) + q_i\varepsilon_i + \left(q_i + \frac{\overline{q}^2}{\sigma_Z^2(\rho\overline{\eta})^2}\right)X - \frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}}Z\right). \tag{A8}$$

Plugging Eq. (A8) and the expression of P, Eq. (16), into g_i, we have

$$g_i = \left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)\left(1 - \frac{h_i}{H}\right) + q_i\varepsilon_i - \frac{1}{\sigma_X^2}\left(1 - \frac{h_i}{H}\right)X + \frac{\rho\overline{\eta}}{\overline{q}}\left(h_i - \frac{\overline{q}^2}{\sigma_Z^2\rho^2\overline{\eta}^2} - \frac{h_i}{H\sigma_X^2}\right)Z. \tag{A9}$$

Then we calculate the expectation and variance of g_i as

$$E_1(g_i) = \frac{h_i}{H}\rho\overline{\eta}\mu_Z, \tag{A10}$$

$$Var(g_i) = \frac{h_i^2}{H^2}\left(\frac{1}{\sigma_X^2} + \frac{\rho^2\overline{\eta}^2}{\overline{q}^2}\sigma_Z^2\left(H - \frac{1}{\sigma_X^2}\right)^2\right)) - h_i = \frac{h_i^2}{H^2}\left(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\right) - h_i. \tag{A11}$$

At the optimal investment strategy, the institution's expected utility under the utility function Eq. (1) is

$$\begin{aligned} U_i &= -E_1\left(\frac{1}{\rho}\ln\left(-\underset{\theta_i}{max}\ E_2(-\exp(-\rho C_i)|Y_i, P)\right)\right) \\ &= -E_1\left[\frac{1}{\rho}\ln\left(\exp(-\rho a_i - \rho\eta_i + \rho k(q_i))\left(1 - 2\rho\gamma_i\frac{1}{h_i}\right)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\left(\hat{\mu}_{Xi} - P\right)^2 h_i\right)\right)\right] \\ &= a_i + \eta_i - k(q_i) + \frac{1}{2\rho}(\ln(h_i - 2\rho\gamma_i) - \ln(h_i)) + \frac{1}{2\rho h_i}E_1\left[(E_1(g_i))^2 + Var(g_i)\right] \\ &= a_i + \eta_i - k(q_i) + \frac{1}{2\rho}(\ln(h_i - 2\rho\gamma_i) - \ln(h_i)) + \frac{h_i}{2\rho H^2}\left(\rho^2\overline{\eta}^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + \overline{q}\right) - \frac{1}{2\rho}\ . \end{aligned}$$

Taking partial derivatives to U_i with respect to q_i and setting them equal to zero, we have Eq. (18). □

Proof of Theorem 4 When the average penalty intensity $\overline{\gamma}_Q$ stays constant, taking partial derivatives to both sides of Eq. (18) with respect to γ_i gives

$$2\rho\frac{\partial k'(\hat{q}_i)}{\partial \hat{q}_i}\frac{\partial \hat{q}_i}{\partial \gamma_i} = -(Q+\hat{q}_i-2\rho\gamma_i)^{-2}\left(\frac{\partial \hat{q}_i}{\partial \gamma_i}-2\rho\right)+(Q+\hat{q}_i)^{-2}\frac{\partial \hat{q}_i}{\partial \gamma_i},$$

And simplifying it delivers

$$\frac{\partial \hat{q}_i}{\partial \gamma_i} = \frac{2\rho(Q+\hat{q}_i-2\rho\gamma_i)^{-2}}{2\rho\frac{\partial k'(\hat{q}_i)}{\partial \hat{q}_i}+(Q+\hat{q}_i-2\rho\gamma_i)^{-2}-(Q+\hat{q}_i)^{-2}} > 0. \tag{A12}$$

Therefore, for a single institution under the *QB* contract, its acquired information's precision increases with the penalty intensity in its contract.

Under the *LB* contract, $\hat{q}_i^L$ satisfies $k'\left(\hat{q}_i^L\right) = \frac{1}{2\rho}\left(\frac{1}{H^2}\left(\rho^2\left(\sigma_Z^2+\left(\mu_Z-\overline{\gamma}_L\right)^2\right)+H+\overline{q}\right)\right)$). When $\mu_Z \geq \overline{\gamma}_L$,

$$k'(\hat{q}_i)-k'\left(\hat{q}_i^L\right) = \frac{1}{2\rho}\left(\frac{1}{Q+\hat{q}_i-2\rho\gamma_i}-\frac{1}{Q+\hat{q}_i^L}+\frac{1}{H^2}\rho^2\left(\mu_Z^2-\left(\mu_Z-\overline{\gamma}_L\right)^2\right)\right) > 0. \tag{A13}$$

Since $\frac{\partial k'(q_i)}{\partial q_i} > 0$, thus $\hat{q}_i > \hat{q}_i^L$.

Because $\hat{q}_i^L$ is independent of γ_i and $\frac{\partial \hat{q}_i}{\partial \gamma_i} > 0$, $\frac{\partial\left(\hat{q}_i-\hat{q}_i^L\right)}{\partial \gamma_i} > 0$. For institutions not subject to performance-based contracts, $\gamma_i = 0$ and its acquired information's precision is $\hat{q}_i^N$, so when $\gamma_i \neq 0$, we have $\hat{q}_i > \hat{q}_i^N$. Because $\hat{q}_i^N$ is independent of γ_i and $\frac{\partial \hat{q}_i}{\partial \gamma_i} > 0$, $\frac{\partial\left(\hat{q}_i-\hat{q}_i^N\right)}{\partial \gamma_i} > 0$.

Then we prove that $\overline{q}$ is the unique solution to $\overline{q} = \int_0^{\lambda_Q} \hat{q}_i^{BI}(\overline{q})di + \int_{\lambda_Q}^1 \hat{q}_i^{NI}(\overline{q})di = \int_0^1 \hat{q}_i(\overline{q})di$. We first prove the existence of $\overline{q}$, and then prove its uniqueness. Plugging $\hat{q}_i$ that satisfies Eq. (15) into $\overline{q} = \int_0^{\lambda_Q} q_i^{BI}di + \int_{\lambda_Q}^1 q_i^{NI}di$, we get the average information precision as

$$\overline{q} = \int_0^{\lambda_Q} \hat{q}_i^{BI}(\overline{q})di + \int_{\lambda_Q}^1 \hat{q}_i^{NI}(\overline{q})di = \int_0^1 \hat{q}_i(\overline{q})di.$$

Let $m(\overline{q}) = \overline{q} - \int_0^1 \hat{q}_i(\overline{q})di$, then $\overline{q}$ is determined by $m(\overline{q}) = 0$. Take partial derivative to both sides of Eq. (18) with respect to $\overline{q}$, and it is easy to show $\frac{\partial \hat{q}_i(\overline{q})}{\partial \overline{q}} < 0$, so $m'(\overline{q}) = 1 - \int_0^1 \frac{\partial \hat{q}_i(\overline{q})}{\partial \overline{q}} di > 0$. Note $k'(\hat{q}_i) = \frac{1}{2\rho}\left(\frac{1}{Q+\hat{q}_i-2\rho\gamma_i} - \frac{1}{Q+\hat{q}_i} + H^{-2}\left(\rho^2\overline{\eta}^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + \overline{q}\right)\right)$. When $\overline{q} = +\infty$, $Q = +\infty$, $H = +\infty$, $H^{-2}\left(\rho^2\overline{\eta}^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + \overline{q}\right) = 0$, $k'(\hat{q}_i) = \frac{1}{2\rho}\left(\frac{1}{Q+\hat{q}_i-2\rho\gamma_i} - \frac{1}{Q+\hat{q}_i}\right) = 0$, therefore $\hat{q}_i(+\infty) = 0$, and $m(+\infty) = +\infty - \int_0^1 0 di = +\infty$.

Since $m'(\overline{q}) > 0$, $m(+\infty) = +\infty$, and $m(0) \leq 0$, so $\overline{q}$ is the unique solution to $\overline{q} = \int_0^{\lambda_Q} \hat{q}_i^{BI}(\overline{q}) di + \int_{\lambda_Q}^1 \hat{q}_i^{NI}(\overline{q}) di = \int_0^1 \hat{q}_i(\overline{q}) di$.

Last, we show how average penalty intensity $\overline{\gamma}_Q$ affects information precision and average information precision. As $\overline{\gamma}_Q$ changes, taking partial derivatives to both sides of Eq. (18) with respect to $\overline{\gamma}_Q$ gives

$$\begin{aligned}
2\rho \frac{\partial k'(\hat{q}_i)}{\partial \hat{q}_i} \frac{\partial \hat{q}_i}{\partial \overline{\gamma}_Q} &= \left(-(Q + \hat{q}_i - 2\rho\gamma_i)^{-2} + \left(Q + \hat{q}_i\right)^{-2}\right)\left(\frac{\partial \hat{q}_i}{\partial \overline{\gamma}_Q} + \frac{2\overline{q}}{\rho^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}\right) \\
&+ 2(Q + \hat{q}_i - 2\rho\gamma_i)^{-2}\rho\frac{\partial \gamma_i}{\partial \overline{\gamma}_Q} \\
&- 2\left(\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + \overline{q}\right)H^{-3}\left(\frac{2\overline{q}}{\rho^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q} + \frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}\right) \\
&+ H^{-2}\left(\frac{2\overline{q}}{\rho^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q} + 2\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}\right).
\end{aligned} \tag{A14}$$

Simplifying it yields

$$\begin{aligned}
&\frac{\partial \gamma_i}{\partial \overline{\gamma}_Q}\frac{2\rho}{(Q + \hat{q}_i - 2\rho\gamma_i)^2} \\
&= \underbrace{\left(2\rho\frac{\partial k'(\hat{q}_i)}{\partial \hat{q}_i} + (Q + \hat{q}_i - 2\rho\gamma_i)^{-2} - \left(Q + \hat{q}_i\right)^{-2}\right)}_{>0}\frac{\partial \hat{q}_i}{\partial \overline{\gamma}_Q} \\
&\quad + \underbrace{H^{-3}\left(2\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + 2\overline{q}\right)}_{>0} \cdot \frac{\partial \overline{q}}{\partial \overline{\gamma}_Q} \\
&\quad + \underbrace{\left[\left((Q + \hat{q}_i - 2\rho\gamma_i)^{-2} - (Q + \hat{q}_i)^{-2} + H^{-3}\left(2\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + 2\overline{q}\right)\right)\frac{2\overline{q}}{\rho^2\sigma_Z^2}\right]}_{>0} \cdot \frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}.
\end{aligned} \tag{A15}$$

The sign of the right-hand side of (Eq. A15) depends on the signs of $\frac{\partial \hat{q}_i}{\partial \overline{\gamma}_Q}$ and $\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}$. Equation (A15) holds for all institutions, including those that increase penalty intensity in the *QB* contract and those un-benchmarked. Now we'll prove $\partial\overline{q}/\partial\overline{\gamma}_Q > 0$

via *apagoge*, or proof by contradiction. The process is first to assume $\partial\overline{q}/\partial\overline{\gamma}_Q \leq 0$, then we look for a counterexample that presents a contradiction, which would prove that $\partial\overline{q}/\partial\overline{\gamma}_Q \leq 0$ cannot be true so that $\partial\overline{q}/\partial\overline{\gamma}_Q > 0$. We find a counterexample involving two types of institutions. The specific proof is as follows.

Assume $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} \leq 0$ for now. When $\overline{\gamma}_Q$ rises, $\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q} > 0$ holds for institutions raising penalty intensity by definition. Since the left-hand side of Eq. (A15) $\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q}\frac{2\rho}{(Q+\hat{q}_i-2\rho\gamma_i)^2} > 0$, to make its right-hand side positive as well, we must have $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} > 0$ given the assumption $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} \leq 0$. For un-benchmarked institutions, $\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q} = 0$. Now the left-hand side of (Eq. A15) is zero, so its right-hand side must be zero as well. Since $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} \leq 0$ as assumed, $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} \geq 0$ is required for un-benchmarked institutions to make the right-hand side of (Eq. A15) equal to zero.

In short, for institutions raising penalty intensity, $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} > 0$ so that $\int\limits_0^{\lambda_Q} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di > 0$. For un-benchmarked institutions, $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} \geq 0$ so that $\int\limits_{\lambda_Q}^{1} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di \geq 0$. Thus, $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} = \int\limits_0^{\lambda_Q} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di + \int\limits_{\lambda_Q}^{1} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di > 0$, which contradicts the assumption $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} \leq 0$. Therefore, $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} > 0$ is proved.

Then we'll prove that as $\overline{\gamma}_Q$ increases, for institutions that raise penalty intensity, $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} > 0$, and for institutions that decrease or do not change penalty intensity, or are un-benchmarked, $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} < 0$. Assume the proportion of the former group in the market is Γ, then the proportion of the latter group is $1-\Gamma$. As $\overline{\gamma}_Q$ increases, $\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q} \leq 0$ holds for the latter group by definition. Now the left-hand side of (Eq. A15) $\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q}\frac{2\rho}{(Q+\hat{q}_i-2\rho\gamma_i)^2} \leq 0$. To make its right-hand side non-positive, since $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} > 0$ as just proved above, we must have $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} < 0$ and thus $\int\limits_{\Gamma}^{1} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di < 0$. Further, to keep $\frac{\partial\overline{\gamma}}{\partial\overline{\gamma}_Q} = \int\limits_0^{\Gamma} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di + \int\limits_{\Gamma}^{1} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di > 0$, $\int\limits_0^{\Gamma} \frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q}di > 0$ must be true, and thus $\frac{\partial\hat{q}_i}{\partial\overline{\gamma}_Q} > 0$ for institutions that raise penalty intensity. □

Proof of Theorem 5 We substitute the expressions of $\hat{\mu}_{Xi}$, h_i, and P into $\hat{\theta}_i$:

$$\begin{aligned}\hat{\theta}_i &= h_i\frac{\hat{\mu}_{Xi} - P}{\rho}\\ &= \frac{1}{\rho}\left(\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\overline{q}}{\sigma_Z^2\rho}\right)\left(1 - \frac{h_i}{H}\right) + q_i\varepsilon_i - \frac{1}{\sigma_X^2}\left(1 - \frac{h_i}{H}\right)X + \frac{\rho}{\overline{q}}\left(h_i - \frac{\overline{q}^2}{\sigma_Z^2\rho^2} - \frac{h_i}{H\sigma_X^2}\right)Z\right).\end{aligned}$$

As the optimal investment amount is a random variable following a normal distribution, we explore how its mean, $E\left(\hat{\theta}_i\right) = \frac{h_i}{H}\mu_Z$, varies with the strength of nonlinear benchmarking. Because of the very large number of (i.e., continuous) institutional investors in the market, changing the benchmarking/penalty intensity of a single institution does not affect aggregate informational efficiency, so $\frac{\partial H}{\partial \gamma_i} = 0$. Due to $\frac{\partial h_i}{\partial \gamma_i} > 0$, $\frac{\partial E\left(\hat{\theta}_i\right)}{\partial \gamma_i} = \frac{\mu_Z}{H}\frac{\partial h_i}{\partial \gamma_i} > 0$. Since

$$E(P) = \frac{1}{H}\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \overline{q}}{\sigma_Z^2 \rho}\right) + \frac{1}{H}\left(H - \frac{1}{\sigma_X^2}\right)\mu_X - \frac{1}{H}\left(\frac{\overline{q}}{\sigma_Z^2 \rho} + \rho\right)\mu_Z = \mu_X - \frac{1}{H}\rho\mu_Z,$$
$$E(X - P) = \frac{1}{H}\rho\mu_Z,$$

$$\begin{aligned} Var(X - P) &= Var\left(-\frac{1}{H}\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \overline{q}}{\sigma_Z^2 \rho}\right) + \frac{1}{H\sigma_X^2}X + \frac{1}{H}\left(\frac{\overline{q}}{\sigma_Z^2 \rho} + \rho\right)Z\right) \\ &= \frac{1}{H^2}\left(\left(\frac{\overline{q}}{\sigma_Z^2 \rho} + \rho\right)^2 \sigma_Z^2 + \frac{1}{\sigma_X^2}\right) = \frac{1}{H^2}\left(\sigma_Z^2\rho^2 + \overline{q} + H\right), \end{aligned}$$

We thus have

$$\frac{\partial E(P)}{\partial \overline{\gamma}_Q} = \frac{\rho\mu_Z}{H^2}\frac{\partial H}{\partial \overline{\gamma}_Q} > 0, \quad \frac{\partial E(X - P)}{\partial \overline{\gamma}_Q} = -\frac{\rho\mu_Z}{H^2}\frac{\partial H}{\partial \overline{\gamma}_Q} < 0$$

$$\frac{\partial Var(X - P)}{\partial \overline{\gamma}_Q} = -\frac{1}{H^3}\underbrace{\left[\left(2\sigma_Z^2\rho^2 + 2\overline{q} + H\right)\frac{2\overline{q}}{\sigma_Z^2\rho^2} + 2\sigma_Z^2\rho^2 + 2\overline{q}\right]}_{>0} \cdot \underbrace{\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}}_{>0} < 0.$$

□

Proof of Theorem 6 Taking partial derivative to both sides of Eq. (A18) with respect to $\overline{\eta}$ gives

$$\begin{aligned} 2\rho\frac{\partial k'\left(\hat{q}_i\right)}{\partial \hat{q}_i}\frac{\partial \hat{q}_i}{\partial \overline{\eta}} &= \left(-(Q + \hat{q}_i - 2\rho\gamma_i)^{-2} + \left(Q + \hat{q}_i\right)^{-2}\right)\left(\frac{\partial \hat{q}_i}{\partial \overline{\eta}} + \frac{2\overline{q}}{\rho^2\overline{\eta}^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\eta}} - 2\frac{\overline{q}^2\overline{\eta}^{-3}}{\rho^2\sigma_Z^2}\right) \\ &\quad + H^{-2}\frac{\partial \overline{q}}{\partial \overline{\eta}} + 2\rho^2\overline{\eta}\left(\sigma_Z^2 + \mu_Z^2\right)H^{-2} \\ &\quad - \left(2\rho^2\overline{\eta}^2\left(\sigma_Z^2 + \mu_Z^2\right) + H + 2\overline{q}\right)H^{-3}\left(\frac{2\overline{q}}{\rho^2\overline{\eta}^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\eta}} + \frac{\partial \overline{q}}{\partial \overline{\eta}} - 2\frac{\overline{q}^2\overline{\eta}^{-3}}{\rho^2\sigma_Z^2}\right). \end{aligned} \quad \text{(A16)}$$

Rearranging (Eq. A16) leads to

$$\begin{aligned} &\underbrace{\left(2\rho\frac{\partial k'\left(\hat{q}_i\right)}{\partial \hat{q}_i} + (Q + \hat{q}_i - 2\rho\gamma_i)^{-2} - \left(Q + \hat{q}_i\right)^{-2}\right)}_{>0}\frac{\partial \hat{q}_i}{\partial \overline{\eta}} \\ &\quad + \underbrace{\left(\left(Q + \hat{q}_i - 2\rho\gamma_i\right)^{-2} - \left(Q + \hat{q}_i\right)^{-2}\right)}_{>0}\frac{2\overline{q}}{\rho^2\overline{\eta}^2\sigma_Z^2}\frac{\partial \overline{q}}{\partial \overline{\eta}} \end{aligned}$$

$$+\underbrace{\left[\left(2\rho^2\overline{\eta}^2\left(\sigma_Z^2+\mu_Z^2\right)+H+2\overline{q}\right)H^{-3}\left(\frac{2\overline{q}}{\rho^2\overline{\eta}^2\sigma_Z^2}+1\right)-H^{-2}\right]}_{>0}\frac{\partial\overline{q}}{\partial\overline{\eta}}$$

$$=\big[(Q+\hat{q}_i-2\rho\gamma_i)^{-2}-\big(Q+\hat{q}_i)^{-2}+\left(2\rho^2\overline{\eta}^2\left(\sigma_Z^2+\mu_Z^2\right)+H+2\overline{q}\right)H^{-3}\big]2$$

$$\frac{\overline{q}^2\overline{\eta}^{-3}}{\rho^2\sigma_Z^2}+2\rho^2\overline{\eta}\left(\sigma_Z^2+\mu_Z^2\right)H^{-2}>0 \tag{A17}$$

Taking partial derivative to both sides of $\overline{q}=\int_0^1\hat{q}_i di$ with respect to $\overline{\eta}$ gives $\frac{\partial\overline{q}}{\partial\overline{\eta}}=\int_0^1\frac{\partial\hat{q}_i}{\partial\overline{\eta}}di$, so $\frac{\partial\overline{q}}{\partial\overline{\eta}}$ and $\frac{\partial\hat{q}_i}{\partial\overline{\eta}}$ should have the same sign. Since the right-hand side of Eq. (A17) is positive, its left-hand side must be positive. The sign of the left-hand side depends on $\frac{\partial\overline{q}}{\partial\overline{\eta}}$ and $\frac{\partial\hat{q}_i}{\partial\overline{\eta}}$, which share the same sign. Thus, to make the left-hand side of Eq. (A17) positive, there must be $\frac{\partial\hat{q}_i}{\partial\overline{\eta}}>0$ and $\frac{\partial\overline{q}}{\partial\overline{\eta}}>0$. It is straightforward to show $\frac{\partial q_i}{\partial\eta_i}=0$ when $\overline{\eta}$ is unchanged. □

Proof of Theorem 7 Taking derivative with respect to $\overline{\eta}$ to price informativeness gives

$$\frac{\partial Q}{\partial\overline{\eta}}=\frac{\partial}{\partial\overline{\eta}}\left(\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}\right)=\frac{2\overline{q}}{\rho^2\overline{\eta}^2\sigma_Z^2}\frac{\partial\overline{q}}{\partial\overline{\eta}}-\frac{2\overline{q}^2}{\rho^2\overline{\eta}^3\sigma_Z^2}=\frac{2\overline{q}^2}{\rho^2\overline{\eta}^3\sigma_Z^2}\left(\frac{\partial\overline{q}}{\partial\overline{\eta}}\frac{\overline{\eta}}{\overline{q}}-1\right). \tag{A18}$$

As the average incentive slope increases, if $\frac{\partial\overline{q}}{\partial\overline{\eta}}\frac{\overline{\eta}}{\overline{q}}>1$, price informativeness rises; if $\frac{\partial\overline{q}}{\partial\overline{\eta}}\frac{\overline{\eta}}{\overline{q}}<1$, price informativeness declines. □

Proof of Theorem 8 We plug the expressions of $\hat{\mu}_{Xi}$, h_i, and P into $\hat{\theta}_i$:

$$E\left(\hat{\theta}_i\right)=\frac{1}{\rho\eta_i}E\left(\left(\hat{\mu}_{Xi}-P\right)h_i\right)=\frac{\overline{\eta}\mu_Z}{\eta_i H}h_i. \tag{A19}$$

Find the partial derivative of $E\left(\hat{\theta}_i\right)$ with respect to the incentive slope η_i:

$$\frac{\partial E\left(\hat{\theta}_i\right)}{\partial\eta_i}=\frac{\overline{\eta}\mu_Z}{H}\left(\frac{\partial h_i}{\partial\eta_i}\eta_i-h_i\right)\eta_i^{-2}=\frac{\overline{\eta}\mu_Z h_i}{H\eta_i^2}\left(\frac{\partial h_i}{\partial\eta_i}\frac{\eta_i}{h_i}-1\right). \tag{A20}$$

Since $\frac{\partial h_i}{\partial\eta_i}=0$, $\frac{\partial E\left(\hat{\theta}_i\right)}{\partial\eta_i}<0$. Therefore, when the market average incentive slope remains unchanged, as an individual incentive slope increases, the institutional investor tends to reduce its risky investment.

$$E(P)=\frac{1}{H}\left(\frac{\mu_X}{\sigma_X^2}+\frac{\mu_Z\overline{q}}{\sigma_Z^2\rho\overline{\eta}}\right)+\frac{1}{H}\left(H-\frac{1}{\sigma_X^2}\right)\mu_X-\frac{1}{H}\left(\frac{\overline{q}}{\sigma_Z^2\rho\overline{\eta}}+\rho\overline{\eta}\right)\mu_Z=\mu_X-\frac{1}{H}\rho\overline{\eta}\mu_Z. \tag{A21}$$

Finding the partial derivative of $E(P)$ with respect to the average incentive slope $\overline{\eta}$, we have $\frac{\partial E(P)}{\partial \overline{\eta}} = \frac{\rho \mu_Z}{H}\left(\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} - 1\right)$. Therefore, $\frac{\partial E(P)}{\partial \overline{\eta}} > 0$ when $\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} > 1$, and $\frac{\partial E(P)}{\partial \overline{\eta}} < 0$ when $\frac{\partial H}{\partial \overline{\eta}}\frac{\overline{\eta}}{H} < 1$. □

Proof of Theorem 9

$$U_P(\eta_i, \gamma_i) = (1 - \eta_i) - a(\eta_i, \gamma_i) + \frac{1 - \eta_i}{\rho \eta_i} E\big(\big(\hat{\mu}_{Xi} - P\big) h_i (X - P)\big) + \gamma_i E\big((X - P)^2\big). \tag{A22}$$

Assuming the participation constraint (26) is binding (i.e., making it an equation) and plugging it into (A22), we can get (27), where $E\big(\big(\hat{\mu}_{Xi} - P\big)(X - P)\big)$ and $E(\big(X - P)^2\big)$ are given in Eqs. (28) and (29) respectively.

Finding the partial derivative of $U_P(\eta_i, \gamma_i)$ with respect to η_i gives Eq. (30).

When $E\big(\big(\hat{\mu}_{Xi} - P\big)(X - P)\big) < 0$, $\frac{\partial U_P(\eta_i, \gamma_i)}{\partial \eta_i} > 0$, and the optimal incentive slope $\hat{\eta}_i = 1$. Taking partial derivative of U_P with respect to γ_i, and plugging $\hat{\eta}_i = 1$ into $\partial U_P / \partial \gamma_i$, we can get:

$$\begin{aligned} \frac{\partial U_P\left(\eta_i, \gamma_i\right)}{\partial \gamma_i} &= E\Big[\Big(X - P\Big)^2\Big] - k'\left(\hat{q}_i\right) \frac{\partial h_i}{\partial \gamma_i} + \frac{A}{2\rho}\frac{\partial h_i}{\partial \gamma_i} \\ &\quad - \frac{1}{2\rho}\left[\frac{1}{h_i}\frac{\partial h_i}{\partial \gamma_i} - \frac{1}{h_i - 2\rho\gamma_i}\left(\frac{\partial h_i}{\partial \gamma_i} - 2\rho\right)\right]. \end{aligned} \tag{A23}$$

Plug $h_i = \frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2 \overline{\eta}^2 \sigma_Z^2} + \hat{q}_i$, Eq. (18), and $\frac{\partial h_i}{\partial \gamma_i} = \frac{(h_i - 2\rho\gamma_i)^{-2}}{1 + \frac{1}{2\rho}[(h_i - 2\rho\gamma_i)^{-2} - h_i^{-2}]}$ into the first order condition $\frac{\partial U_P(\eta_i, \gamma_i)}{\partial \gamma_i} = 0$, then $\hat{\gamma}_i$ satisfies Eq. (31).

When $E\big(\big(\hat{\mu}_{Xi} - P\big)(X - P)\big) > 0$, $\frac{\partial U_P(\eta_i, \gamma_i)}{\partial \eta_i} < 0$. From Eq. (27), we know $\eta_i \neq 0$ and $\eta_i \in (0, 1]$. Therefore, if the investor delegates investment to the institution, the optimal incentive slope does not exist. The smaller η_i is, as it approaches 0, the greater the investor's expected utility.

When $\eta_i = 0$, let $y = X|Y_i, P$. Recalculate the institution's expected utility:

$$\begin{aligned} U_i(\gamma_i) &= -E_1\left[\frac{1}{\rho}\ln(-E_2(-\exp(-\rho(\omega_i - k(q_i)))|Y_i, P))\right] \\ &= -E_1\left[\frac{1}{\rho}\ln\Big(-E_2\Big(-\exp\Big(-\rho a(0, \gamma_i) + \rho\gamma_i(y - P)^2 + \rho k(q_i)\Big)\Big)\Big)\right] \\ &= -E_1\Bigg[\frac{1}{\rho}\ln\Big(E_2\Big(\exp\Big(-\rho a(0, \gamma_i) + \rho k(q_i) + \rho\gamma_i(y - \hat{\mu}_{Xi})^2 \\ &\quad + 2\rho\gamma_i(y - \hat{\mu}_{Xi})(\hat{\mu}_{Xi} - P) + \rho\gamma_i(\hat{\mu}_{Xi} - P)^2\Big)\Big)\Big)\Bigg] \end{aligned}$$

and its rearrangement gives.

$$U_i(\gamma_i) = E_1\left[a(0,\gamma_i) - k(q_i) + \frac{1}{2\rho}\ln\left(\frac{h_i - 2\rho\gamma_i}{h_i}\right) - \frac{\gamma_i}{h_i - 2\rho\gamma_i}(\hat{\mu}_{Xi} - P)^2 h_i\right].$$

Based on Eqs. (A10) and (Eq. A11), we have

$$\begin{aligned} &E_1\left[\frac{\gamma_i(\hat{\mu}_{Xi} - P)^2 h_i}{h_i - 2\rho\gamma_i}\right] \\ &\quad = \frac{1}{h_i}\frac{\gamma_i}{h_i - 2\rho\gamma_i}E_1\left[\left(\left(\hat{\mu}_{Xi} - P\right)h_i\right)^2\right] \\ &\quad = \frac{\gamma_i}{h_i - 2\rho\gamma_i}\left[\frac{h_i}{H^2}\left(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\right) - 1 + \left(\frac{\rho\overline{\eta}\mu_Z}{H}\right)^2 h_i\right], \end{aligned}$$

Let $B = \frac{1}{H^2}\left(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\right) + \left(\frac{\rho\overline{\eta}\mu_Z}{H}\right)^2$, thus

$$U_i(\gamma_i) = a(0,\gamma_i) - k(q_i) + \frac{1}{2\rho}ln\left(\frac{h_i - 2\rho\gamma_i}{h_i}\right) - \frac{\gamma_i}{h_i - 2\rho\gamma_i}[Bh_i - 1]. \quad \text{(A24)}$$

Find the first order condition of $U_i(\gamma_i)$ with respect to q_i and set it as zero, then we have Eq. (32).

Now the investor's expected utility is

$$\begin{aligned} U_P(\gamma_i) &= E_1[E_2((W_i - \omega_i)|Y_i, P)] \\ &= E_1\left[E_2\left[\left(1 + \theta_i(X - P) + \gamma_i(X - P)^2 - a(\eta_i, \gamma_i)\right)|Y_i, P\right]\right] \\ &= E_1\left[1 + \theta_i\left(\hat{\mu}_{Xi} - P\right) + \gamma_i\left(\hat{\mu}_{Xi} - P\right)^2 + \frac{\gamma_i}{h_i} - u_{0i} - \frac{1}{2\rho}\ln\left(\frac{h_i}{h_i - 2\rho\gamma_i}\right) - k(q_i) + \frac{1}{2\rho}Ah_i - \frac{1}{2\rho}\right] \end{aligned}$$

Taking partial derivative to $U_P(\gamma_i)$ with respect to γ_i and setting it equal to zero, we get Eq. (33). □

Appendix B: Proof of selected Theorems under CARA expected utility function

Breugem and Buss (2019) demonstrate that under the CARA utility, linear benchmarking cannot motivate institutions to obtain information, and increasing the proportion of benchmarked institutions will not affect price informativeness. In this appendix, we will show that under the CARA utility, the nonlinear benchmark can still effectively motivate institutions to acquire information, enhance price informativeness, and reduce the volatility of returns. That is, the results regarding the impact of nonlinear benchmarked contracts are robust.

With a CARA utility function, the institution's expected utility is

$$\tilde{U}_i = E_1\big(E_2\big[-\exp(-\rho C_i)|Y_i, P\big]\big), \tag{B1}$$

Consistent with the proofs of Theorems 1 and 2, the institution's investment strategy and the risky asset price under a CARA utility satisfy Eqs. (15) and (16) respectively. Next, we will prove that under the CARA utility, the conclusions pertaining to the QB contract's impact on the institution's information acquisition, price informativeness and price fluctuation still hold. We first prove Theorems 4 and 5 under CARA expected utility function.

$$\begin{aligned}
&\text{Let } \ Z_i = \sqrt{h_i}\big(\hat{\mu}_{Xi} - P\big).\ \text{ Since } \ Z_i \sim N(E_1(Z_i), Var_1(Z_i))\\
&E_1\left(\exp\left(-\frac{1}{2}\big(\hat{\mu}_{Xi} - P\big)^2 h_i\right)\right)\\
&\quad = E_1\left(\exp\left(-\frac{1}{2}(Z_i - E_1(Z_i))^2 - (Z_i - E_1(Z_i))E_1(Z_i) - \frac{1}{2}(E_1(Z_i))^2\right)\right)\\
&\quad = (1 - 2Var_1(Z_i))(-\frac{1}{2})^{-\frac{1}{2}}\ \exp\left[\frac{1}{2}(-E_1(Z_i))^2\left(1 - 2Var_1(Z_i)\left(-\frac{1}{2}\right)\right)^{-1} Var_1(Z_i) - \frac{1}{2}(E_1(Z_i))^2\right]\\
&\quad = (1 + Var_1(Z_i))^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\frac{(E_1(Z_i))^2}{1 + Var_1(Z_i)}\right) = \left(\left(1 + \frac{Var_1(g_i)}{h_i}\right)\exp\left(\frac{(E_1(g_i))^2}{h_i + Var_1(g_i)}\right)\right)^{-\frac{1}{2}}.
\end{aligned}$$

Plugging Eqs. (A10) and (A11) into the equation above and simplifying it yield

$$E_1\left(\exp\left(-\frac{1}{2}\big(\hat{\mu}_{Xi} - P\big)^2 h_i\right)\right) = \left(\frac{h_i}{H^2}\big(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\big)\exp\left(\rho^2\overline{\eta}^2\mu_Z^2\big(H + \rho^2\overline{\eta}^2\sigma_Z^2 + \overline{q}\big)^{-1}\right)\right)^{-\frac{1}{2}} \tag{B2}$$

At the optimal investment strategy, the institution's expected utility under the utility function Eq. (B1) is

$$\begin{aligned}
\tilde{U}_i &= E_1\left(\underset{\theta_i}{max}\ E_2(-\exp(-\rho C_i)|Y_i, P)\right)\\
&= -\left(\left(1 - 2\rho\gamma_i\frac{1}{h_i}\right)^{-\frac{1}{2}}\exp(-\rho a_i - \rho\eta_i + \rho k(q_i))\right)E_1\left[\exp\left(-\frac{1}{2}\big(\hat{\mu}_{Xi} - P\big)^2 h_i\right)\right]\\
&= -\left(1 - 2\rho\gamma_i\frac{1}{h_i}\right)^{-\frac{1}{2}}\exp(-\rho a_i - \rho\eta_i + \rho k(q_i))\\
&\quad\cdot\left[\frac{h_i}{H^2}\big(H + (\rho\overline{\eta})^2\sigma_Z^2 + \overline{q}\big)\exp\left((\rho\overline{\eta})^2\mu_Z^2\big(H + (\rho\overline{\eta})^2\sigma_Z^2 + \overline{q}\big)^{-1}\right)\right]^{-\frac{1}{2}}.
\end{aligned}$$

Taking partial derivatives to $\tilde{U}_i$ with respect to q_i and setting them equal to zero, we have

$$k'(\tilde{q}_i) = \frac{1}{2\rho}\left(\frac{1}{\sigma_X^2} + \frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2} + \tilde{q}_i - 2\rho\gamma_i\right)^{-1}. \tag{B3}$$

Differentiate both sides of Eq. (B3) with respect to γ_i,

$$2\rho\frac{\partial k'(\tilde{q}_i)}{\partial\tilde{q}_i}\frac{\partial\tilde{q}_i}{\partial\gamma_i} = -(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\left(\frac{\partial\tilde{q}_i}{\partial\gamma_i}-2\rho\right), \tag{B4}$$

and its simplification gives $\frac{\partial\tilde{q}_i}{\partial\gamma_i} = \frac{2\rho(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}}{2\rho\frac{\partial k'(\tilde{q}_i)}{\partial\tilde{q}_i}+(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}} > 0$.

With linear benchmarking, $\tilde{q}_i^L$ satisfies $k'\left(\tilde{q}_i^L\right) = \frac{1}{2\rho}\frac{1}{Q+\tilde{q}_i^L}$. As $\frac{\partial\tilde{q}_i^L}{\partial\gamma_i^L} = 0$, the institution's acquired information precision is the same as when $\gamma_i^L = 0$. Since $\frac{\partial k'(q_i)}{\partial q_i} > 0$ and $\tilde{q}_i = \tilde{q}_i^L$ when $\gamma_i = \gamma_i^L = 0$, thus $\tilde{q}_i > \tilde{q}_i^L$ when $\gamma_i > 0$. Because $\tilde{q}_i^L$ is independent of γ_i and $\frac{\partial\tilde{q}_i}{\partial\gamma_i} > 0$, thus $\frac{\partial\left(\tilde{q}_i-\tilde{q}_i^L\right)}{\partial\gamma_i} > 0$.

Taking derivative to both sides of Eq. (B3) with respect to $\overline{q}$, we get

$$2\rho\frac{\partial k'(\tilde{q}_i)}{\partial\tilde{q}_i}\frac{\partial\tilde{q}_i}{\partial\overline{q}} = -(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\left(\frac{\partial Q}{\partial\overline{q}}+\frac{\partial\tilde{q}_i}{\partial\overline{q}}\right). \tag{B5}$$

Rearranging (B5) gives

$$\underbrace{\left(2\rho\frac{\partial k'(\tilde{q}_i)}{\partial\tilde{q}_i}+(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\right)}_{>0}\frac{\partial\tilde{q}_i}{\partial\overline{q}} = -\frac{1}{(Q+\tilde{q}_i-2\rho\gamma_i)^2}\frac{2\overline{q}}{\rho^2\sigma_Z^2} < 0,$$

Thus $\frac{\partial\tilde{q}_i}{\partial\overline{q}} < 0$. Construct the function $n(\overline{q}) = \overline{q} - \int\limits_0^1 \tilde{q}_i(\overline{q})di$, then $n'(\overline{q}) = 1 - \int\limits_0^1 \frac{\partial\tilde{q}_i(\overline{q})}{\partial\overline{q}}di > 0$. The proof of the uniqueness of $\overline{q}$ is similar to its proof in Theorem 4, except that $\tilde{q}_i$ as in Eq. (B3) is plugged into $\overline{q} = \int\limits_0^{\lambda_Q} q_i^{BI}di + \int\limits_{\lambda_Q}^1 q_i^{NI}di$. The average information precision $\overline{q}$ thus satisfies

$$\overline{q} = \int\limits_0^{\lambda_Q}\tilde{q}_i^{BI}(\overline{q})di + \int\limits_{\lambda_Q}^1\tilde{q}_i^{NI}(\overline{q})di = \int\limits_0^1\tilde{q}_i(\overline{q})di.$$

The proof of $\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q} > 0$ is similar to that under the utility function U_i. Taking derivative with respect to $\overline{\gamma}_Q$ to both sides of Eq. (B3), we have

$$2\rho\frac{\partial k'(\tilde{q}_i)}{\partial\tilde{q}_i}\frac{\partial\tilde{q}_i}{\partial\overline{\gamma}_Q} = -(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\left(\frac{\partial\tilde{q}_i}{\partial\overline{\gamma}_Q}+\frac{2\overline{q}}{\rho^2\sigma_Z^2}\frac{\partial\overline{q}}{\partial\overline{\gamma}_Q}\right)+2(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\rho\frac{\partial\gamma_i}{\partial\overline{\gamma}_Q}. \tag{B6}$$

Rearranging (B6) by separating items involving $\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}$ and $\frac{\partial \tilde{q}_i}{\partial \overline{\gamma}_Q}$, can lead to a result similar to Eq. (A15):

$$2(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\rho\frac{\partial \gamma_i}{\partial \overline{\gamma}_Q}=\underbrace{\left(2\rho\frac{\partial k'(\tilde{q}_i)}{\partial \tilde{q}_i}+(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\right)}_{>0}\frac{\partial \tilde{q}_i}{\partial \overline{\gamma}_Q}+\underbrace{(Q+\tilde{q}_i-2\rho\gamma_i)^{-2}\frac{2\overline{q}}{\rho^2\sigma_Z^2}}_{>0}\frac{\partial \overline{q}}{\partial \overline{\gamma}_Q}. \tag{B7}$$

Other proof steps and analysis in Theorems 4 and 5 are similar to those under the utility function U_i, so will not be repeated.

Then we prove that the results in Theorem 6 also hold under the CARA expected utility. Taking derivatives with respect to $\overline{\eta}$ to both sides of Eq. (B3), we get

$$2\rho\frac{\partial k'(\tilde{q}_i)}{\partial \tilde{q}_i}\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}=-\left(\frac{1}{\sigma_X^2}+\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}+\tilde{q}_i-2\rho\gamma_i\right)^{-2}\left(\frac{2\rho^2\overline{\eta}^2\sigma_Z^2\overline{q}\frac{\partial \overline{q}}{\partial \overline{\eta}}-2\overline{q}^2\rho^2\sigma_Z^2\overline{\eta}}{(\rho^2\overline{\eta}^2\sigma_Z^2)^2}+\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}\right). \tag{B8}$$

Its rearrangement leads to

$$\underbrace{\left[2\rho\frac{\partial k'(\tilde{q}_i)}{\partial \tilde{q}_i}+\left(\frac{1}{\sigma_X^2}+\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}+\tilde{q}_i-2\rho\gamma_i\right)^{-2}\right]}_{>0}\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}+\underbrace{\left[2\overline{q}\left(\frac{1}{\sigma_X^2}+\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}+\tilde{q}_i-2\rho\gamma_i\right)^{-2}(\rho^2\overline{\eta}^2\sigma_Z^2)^{-1}\right]}_{>0}\frac{\partial \overline{q}}{\partial \overline{\eta}}$$
$$=\underbrace{\left(\frac{1}{\sigma_X^2}+\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}+\tilde{q}_i-2\rho\gamma_i\right)^{-2}\frac{2\overline{q}^2}{\rho^2\sigma_Z^2\overline{\eta}^3}}_{>0}. \tag{B9}$$

Taking derivatives with respect to $\overline{\eta}$ to $\overline{q}=\int_0^1\tilde{q}_i(\overline{q})di$ gives $\frac{\partial \overline{q}}{\partial \overline{\eta}}=\int_0^1\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}di$, so $\frac{\partial \overline{q}}{\partial \overline{\eta}}$ has the same sign as $\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}$. Based on Eq. (B9), we have $\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}>0$ and $\frac{\partial \overline{q}}{\partial \overline{\eta}}>0$. □

Last, we prove that the results in Theorems 7 and 8 hold under the CARA expected utility. Specifically, both price informativeness and the equilibrium price of the risky asset are decreasing in the average incentive slope.

Under the CARA expected utility, since $k'(\tilde{q}_i)=\frac{1}{2\rho}\left(\frac{1}{\sigma_X^2}+\frac{\overline{q}^2}{\rho^2\overline{\eta}^2\sigma_Z^2}+\tilde{q}_i-2\rho\gamma_i\right)^{-1}$ (see Eq. (B3),

$$\frac{\partial \overline{q}}{\partial \overline{\eta}}=\left[(h_i-2\rho\gamma_i)^{-2}\frac{2\overline{q}^2}{\rho^2\sigma_Z^2\overline{\eta}^3}-\left(2\rho\frac{\partial k'(q_i)}{\partial q_i}+(h_i-2\rho\gamma_i)^{-2}\right)\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}\right](2\overline{q}(h_i-2\rho\gamma_i)^{-2}(\rho^2\overline{\eta}^2\sigma_Z^2)^{-1})^{-1},$$

$$\frac{\partial \overline{q}}{\partial \overline{\eta}}\frac{\overline{\eta}}{\overline{q}}=\left[(h_i-2\rho\gamma_i)^{-2}\frac{2\overline{q}}{\rho^2\sigma_Z^2\overline{\eta}^2}-\left(2\rho\frac{\partial k'(q_i)}{\partial q_i}+(h_i-2\rho\gamma_i)^{-2}\right)\frac{\partial \tilde{q}_i}{\partial \overline{\eta}}\frac{\overline{\eta}}{\overline{q}}\right]\left((h_i-2\rho\gamma_i)^{-2}\frac{2\overline{q}}{\rho^2\sigma_Z^2\overline{\eta}^2}\right)^{-1}<1.$$

Thus $\frac{\partial \overline{q}}{\partial \overline{\eta}} \frac{\overline{\eta}}{\overline{q}} < 1$, and $\frac{\partial Q}{\partial \overline{\eta}} < 0$. Price informativeness declines when the average incentive slope rises.

Moreover,

$$
\begin{aligned}
\frac{\partial H}{\partial \overline{\eta}} \frac{\overline{\eta}}{H} &= \frac{\overline{\eta}}{H} \left(\frac{2\overline{q}}{\rho^2 \sigma_Z^2 \overline{\eta}^2} + 1 \right) \frac{\partial \overline{q}}{\partial \overline{\eta}} - \frac{2\overline{q}^2}{H \rho^2 \sigma_Z^2 \overline{\eta}^2} \\
&< \left(\frac{2\overline{q}}{\rho^2 \sigma_Z^2 \overline{\eta}^2} + 1 \right) \left[(h_i - 2\rho\gamma_i)^{-2} \frac{2\overline{q}^2}{H \rho^2 \sigma_Z^2 \overline{\eta}^2} \right] \\
&\quad \left(\frac{2\overline{q}}{\rho^2 \overline{\eta}^2 \sigma_Z^2} (h_i - 2\rho\gamma_i)^{-2} \right)^{-1} - \frac{2\overline{q}^2}{H \rho^2 \sigma_Z^2 \overline{\eta}^2} < 1
\end{aligned}
$$

Therefore, the equilibrium risky asset price is decreasing in the average incentive slope. □

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